

Hedonic Price-Rent Ratios, User Cost, and Departures from Equilibrium in the Housing Market

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Abstract:

Departures from equilibrium in the housing market can be detected by comparing the actual price-rent ratio with the price-rent ratio derived from the user cost equilibrium condition. We consider three ways of improving the applicability of this approach. First, the equilibrium price-rent ratio assumes that the sold and rented dwellings being compared are of equal quality, which is typically not the case. We show how hedonic methods can be used to quality-adjust the actual price-rent ratio. Second, the equilibrium price-rent ratio depends on the unobserved expected capital gain. We explore ways of imputing this value. Third, both the actual and equilibrium price-rent ratios vary over the housing distribution. Hence it is not enough to simply focus on the median. We illustrate our methods using a data set of prices and rents for 730,000 houses in Sydney, Australia over the period 2001 to 2009. In particular, we find that quality adjusting reduces the actual price-rent ratio by on average 18 percent.

Keywords: House price index; Rent index; Quality adjustment; Hedonic imputation; Expected capital gains; Fisher index; Real estate market

JEL Classification Codes: C43; E01; E31; R31

1 Introduction

Recent events have shown how the housing market can impact on the rest of the economy, as a bust in the US housing market precipitated a global financial crisis. As housing markets are particularly prone to booms and busts, it is particularly important that policy makers and other market participants can detect departures from equilibrium before they become too extreme.

One way of detecting such departures is to compare the user cost of owner-occupying with the cost of renting. In equilibrium, households should be indifferent between these alternatives. Departures from equilibrium therefore can be detected by comparing actual price-rent ratios with the price-rent ratio derived from the user-cost equilibrium condition.

Many applications of the user-cost equilibrium condition focus on changes in the price-rent ratio rather than its level. This is because price and rent indexes are easier to obtain than actual prices and rents measured in dollars. For example, Himmelberg, Mayer and Sinai (2005), compare a repeat-sales price index calculated for single-family houses obtained from the Office of Federal Housing Enterprise Oversight (OFHEO) – now part of the Federal Housing Finance Agency (FHFA) – with an index of annual average rents of two-bedroom apartments obtained from REIS (a real estate consulting firm). Gallin (2008) and Campbell, Davis, Gallin and Martin (2009) use the same FHFA repeat-sales price index as Himmelberg et al., and the tenant rent index (part of the rent of shelter index) from the CPI. Duca, Muellbauer and Murphy (2011) compare the FHFA repeat-sales index with the rental fixed dwelling index from the personal consumption expenditure (PCE) price index produced by the Bureau of Economic Analysis.

There are two serious problems in this context with using price and rent indexes. First, as Smith and Smith (2006) point out, there may be inconsistencies between the price and rent indexes:

[T]he dwellings included in price indexes do not match the dwellings in rent indexes, so that the resulting comparison is of apples to oranges. The ratio of a home sale price index to a rent index can rise because the prices of homes in desirable neighborhoods increased more than did the rents of apartment buildings in less desirable neighborhoods. Or perhaps the quality of the average home in the price

index has increased relative to the quality of the average property in the rent index. In any case, gauging fundamental value requires actual rent and sale price data, not indexes with arbitrary scales. (p. 7)

Second, as noted in the final sentence of this quote, using price and rent indexes it is not possible to answer the most fundamental questions which are: (i) whether the price-rent ratio is above or below its equilibrium level and (ii) whether the price-rent ratio is moving towards or away from equilibrium.

To answer these questions we need to compute the actual price-rent ratio in each period, and not just changes in the price-rent ratio. One way of doing this is to compute the ratio of the median dwelling sold to the median dwelling rented. The equilibrium condition, however, assumes that a household is choosing between owner-occupying and renting dwellings of equal quality. In practice, the median sold dwelling tends to be of better quality than the median rented dwelling. Using a data set consisting of 730,000 price and rent observations for Sydney, Australia over the period 2001 to 2009, we find the difference is on average 18 percent.¹ The actual price-rent ratio, therefore, needs to be quality adjusted before it can be compared with its equilibrium counterpart (or the comparison will be biased towards finding that the price-rent ratio is above its equilibrium level).² We show how this can be done using hedonic methods that impute prices for rented dwellings, and impute rents for sold dwelling.

When imputing prices and rents, an important consideration in our data set is missing and omitted characteristics (where a missing characteristic is missing for a particular dwelling while an omitted characteristic is missing for all dwellings). We correct for missing characteristics by estimating multiple versions of our hedonic models, each with a different mix of characteristics, and then impute the price or rent of a dwelling from whichever model has exactly its mix of characteristics. We correct for omitted variables using a sample of dwellings for which we have both price and rent observations.

¹This finding is consistent with the existing literature. For example, according to the American Housing Survey (2001), 82 percent of owner-occupied dwellings are detached single-family homes, while the corresponding figure for rental dwellings is only 23 percent (see also Gallin 2008 and Heston and Nakamura 2009). Given that most sold dwellings end up owner-occupied, a similar pattern should be observed for sold versus rented dwellings.

²It should be noted that the quality difference between sold and rented dwellings in our data set is not stable over time. In the first half of our sample it is greater than 18 percent. By 2009 it has fallen to zero.

We also consider two problems that arise when computing the equilibrium price-rent ratio. First, the expected capital gain on housing - a crucial input into the user-cost equilibrium condition - is not directly observed. While it can be imputed from the past performance of the housing market, we find that the resulting equilibrium price-rent ratio depends critically on the time horizon over which past performance is measured. In particular, when the time horizon is too short the equilibrium price-rent ratio is prone to become volatile and to rise in booms and fall in busts, both of which effects are liable to undermine the method's ability to detect departures from equilibrium. We therefore recommend a long time horizon of 30 years. When expectations are extrapolated over this horizon, we find that the price-rent ratio in Sydney was above its equilibrium level from 2001 to 2008, although not in 2009. In the absence of quality adjustment, the departure from equilibrium seems even larger than it actually was. Alternatively, the expected capital gain can be derived from the user cost equilibrium condition if we assume the market is in equilibrium. Using this approach we find that the expected real capital gain would need to be 4.0 percent per year, which rises to 4.6 percent in the absence of quality adjustment. Compared with other cities 4.0 percent seems too high, thus again leading to the conclusion that the price-rent ratio was above its equilibrium level in Sydney.

A second problem with the equilibrium price-rent ratio is that different conditions may apply in different segments of the market. For example, the depreciation rate may be lower at the high end (where the share of land in the total value of a dwelling tends to be higher). This acts to push up the equilibrium price-rent ratio at the high end. Also, households at the low end may be credit constrained, thus pushing down the equilibrium price-rent ratio at the low end. Empirically we find that the actual price-rent ratio is indeed higher at the high end. More generally, this type of cross-section analysis demonstrates that it is not enough to simply focus on the median. Even if the median price-rent ratio equals the equilibrium price-rent ratio (calculated at the median), this does not necessarily imply that either the high or low ends of the market are in equilibrium.

Beyond this, our methodology and results also have applications that extend beyond the main issues addressed here. For example, failure to account for the quality difference between owner-occupied and rented dwellings and cross-section variation in the price-rent ratio may result in the flow of housing services in national accounts (and hence GDP) being

mismeasured.

The remainder of this paper is structured as follows. Section 2 explains the user-cost equilibrium condition. Section 3 develops our hedonic approach for computing price-rent ratios at the level of individual dwellings. Section 4 describes our data set, and then explains our methods for correcting for missing characteristics and omitted variables. Our estimates of quality bias in actual price-rent ratios are presented in section 5. Section 6 derives equilibrium price-rent ratios from the user-cost equilibrium condition and then checks for departures from equilibrium. Some implications of our findings for the measurement of GDP are considered in section 7. Finally, our conclusions are discussed in section 8.

2 The User-Cost Equilibrium Condition

The user cost of a durable good is the present value of buying it, using it for one period and then selling it (see Hicks 1946). In equilibrium this should equal the cost of renting the good for one period. Following Himmelberg et al. (2005) and Girouard, Kennedy, Noord and André (2006), the equilibrium condition can be written as follows:

$$R_t = u_t P_t, \tag{1}$$

where R_t is the period t rental price, P_t the purchase price, $u_t P_t$ is user cost, and u_t the per dollar user cost. In a housing context, per dollar user cost can be calculated as follows:

$$u_t = r_t + \omega_t + \delta_t + \gamma_t - g_t, \tag{2}$$

where r denotes the risk-free interest rate, ω is the property tax rate, δ the depreciation rate for housing, γ the risk premium of owning as opposed to renting, and g the expected capital gain. That is, an owner occupier foregoes interest on the market value of the dwelling, incurs property taxes and depreciation, incurs risk (mainly due to the inherent uncertainty of future price and rent movements in the housing market) and benefits from any capital gains on the dwelling.³ If $R_t > u_t P_t$, owner-occupying becomes more attractive and hence this should exert upward pressure on P and downward pressure on R until equilibrium is restored. The

³In some countries, owner-occupiers get the benefit from the tax deduction on the mortgage interest payments (see Girouard et al. 2006 for a list of OECD countries providing such benefits). For these countries, r_t should be adjusted to include the offsetting tax benefit. However, no such benefit is provided to the owner occupiers in Australia.

converse argument applies when $R_t < u_t P_t$.

Rearranging (1), we obtain that in equilibrium the price-rent ratio should equal the reciprocal of per dollar user cost (i.e., $P_t/R_t = 1/u_t$). If the actual price-rent ratio exceeds, or is less than, our estimate of the reciprocal of per dollar user cost it follows that the housing market is not in equilibrium.

The equilibrium condition (1), however, implicitly assumes that P_t and R_t are calculated for properties of equivalent quality. If instead P_t refers to dwellings which are of superior quality to the dwellings referred by R_t , then P_t/R_t is overestimated and, as a result, the user cost equilibrium condition will be biased towards finding that the price-rent ratio is above its equilibrium level.

One problem that arises when calculating the equilibrium price-rent ratio is that the expected capital gain g is not directly observable. g can be separated into two components: the expected real capital gain and expected inflation. Of these, the expected real capital gain is more problematic. A standard approach is to assume that the expected real capital gain is extrapolated from the past performance of the housing market. This, however, may cause $1/u$ to fluctuate a lot over time, thus potentially seriously undermining its usefulness (see Verbrugge 2008 and Diewert 2009). Extrapolation may also push up the equilibrium price-rent ratio during a boom, thus potentially undermining the user-cost equilibrium condition's ability to detect a departure from equilibrium. Using our Sydney data set, we explore in section 6 the impact of alternative extrapolation time horizons on the equilibrium price-rent ratio.

Alternatively, the expected capital gain can be imputed from the equilibrium user cost condition. To the best of our knowledge, this approach – first suggested by Diewert (1983) – has never before been applied to data. This is perhaps because it requires price-rent ratios in levels which are difficult to obtain. More specifically, rearranging the user cost formula in (2) and imposing the equilibrium condition in (1) yields the following:

$$g_t = r_t + \omega_t + \delta_t + \gamma_t - \frac{R_t}{P_t}. \quad (3)$$

Setting R_t/P_t equal to the reciprocal of the median quality-adjusted price-rent ratio in period t and inserting estimates of r_t , ω_t , δ_t and γ_t , we obtain an estimate of g_t . If the resulting capital gain g_t is deemed implausible, then we can conclude that the assumption of equilibrium is incorrect.

We can quantify the impact of quality bias on our perception of the housing market by substituting the quality-unadjusted price-rent ratio $[R_t/P_t]_{unadj}$ in place of the quality-adjusted price-rent ratio $[R_t/P_t]_{adj}$ in (3) as follows:

$$G_t = \left[\frac{R_t}{P_t} \right]_{adj} - \left[\frac{R_t}{P_t} \right]_{unadj}. \quad (4)$$

G_t , which does not depend on the chosen values of all the parameters in (3), measures how quality bias affects the expected capital gain required for equilibrium. We apply this approach to our Sydney data in section 6.2.

A second problem with the equilibrium price-rent ratio is that some of the parameters (e.g., depreciation) in (2) may differ for different segments of the market, or that (1) may not hold for all market segments (e.g., when credit constraints are binding). These issues and their implications are explored in section 6.3.

3 Constructing Quality-Adjusted Price-Rent Ratios

3.1 The hedonic imputation method

The hedonic method dates back at least to the 1920s. It was, however, only after Griliches (1961) that hedonic methods started to receive serious attention (see Schultze and Mackie 2002). The hedonic model is a reduced form equation which regresses the price of a product on a vector of characteristics (whose prices are not independently observed).

The hedonic approach can be implemented in a housing context in different ways (see for example Coulson 2008, Silver 2012 and Hill 2013). In our context the most appropriate method is the hedonic imputation method where a separate hedonic model is estimated for each comparison period (typically using a semilog functional form).⁴

$$y_t = X_t \beta_t + u_t, \quad (5)$$

where y_t is an $H_t \times 1$ vector with elements $y_h = \ln p_h$ (where H_t denotes the number of

⁴Alternative functional forms, such as linear or Box-Cox transformations, are sometimes also considered. See Malpezzi (2003) for a discussion of some of the advantages of semilog in a hedonic context, and Diewert, Heravi and Silver (2009) and Rambaldi and Rao (2013) for advantages of the hedonic imputation method. Rambaldi and Rao also show how the stability of the hedonic imputation method can be improved by using Kalman filters to link the hedonic models across time periods.

dwelling sold in period t), X_t is an $H_t \times C$ matrix of characteristics (some of which may be dummy variables), β_t is a $C \times 1$ vector of characteristic shadow prices, and u_t is an $H_t \times 1$ vector of random errors. Examples of characteristics include the number of bedrooms, number of bathrooms, land area, and postcode.

Once the hedonic model has been estimated separately for each period, the prices of dwellings sold in one period can be imputed from the hedonic model of another period. For example, let $\hat{p}_{th}(x_{sh})$ denote the estimated price in period t of a dwelling h sold in period s . This price is imputed by substituting the characteristics of dwelling h into the estimated hedonic model of period t as follows:

$$\hat{p}_{th}(x_{sh}) = \exp\left(\sum_{c=1}^C \hat{\beta}_{ct} x_{csh}\right),$$

where $c = 1, \dots, C$ indexes the set of characteristics included in the hedonic model. A Laspeyres-type hedonic index that compares periods s and t using the dwellings sold in period s can now be constructed in one of two ways:

$$\begin{aligned} \text{L1: } P_{st}^{L1} &= \sum_{h=1}^{H_s} w_{sh} [\hat{p}_{th}(x_{sh})/p_{sh}] = \sum_{h=1}^{H_s} \hat{p}_{th}(x_{sh}) \bigg/ \sum_{h=1}^{H_s} p_{sh} \\ \text{L2: } P_{st}^{L2} &= \sum_{h=1}^{H_s} \hat{w}_{sh} [\hat{p}_{th}(x_{sh})/\hat{p}_{sh}(x_{sh})] = \sum_{h=1}^{H_s} \hat{p}_{th}(x_{sh}) \bigg/ \sum_{h=1}^{H_s} \hat{p}_{sh}(x_{sh}), \end{aligned} \quad (6)$$

where w_{sh} and \hat{w}_{sh} denote actual and imputed expenditure shares calculated as follows:

$$w_{sh} = p_{sh} / \sum_{m=1}^{H_s} p_{sm}, \quad \hat{w}_{sh} = \hat{p}_{sh}(x_{sh}) / \sum_{m=1}^{H_s} \hat{p}_{sm}(x_{sm}).$$

In an analogous manner corresponding Paasche-type hedonic indexes that compare periods s and t using the dwellings sold in period t can be constructed:

$$\begin{aligned} \text{P1: } P_{st}^{P1} &= \left\{ \sum_{h=1}^{H_t} w_{th} [p_{th}/\hat{p}_{sh}(x_{th})]^{-1} \right\}^{-1} = \sum_{h=1}^{H_t} p_{th} \bigg/ \sum_{h=1}^{H_t} \hat{p}_{sh}(x_{th}) \\ \text{P2: } P_{st}^{P2} &= \left\{ \sum_{h=1}^{H_t} \hat{w}_{th} [\hat{p}_{th}(x_{th})/\hat{p}_{sh}(x_{th})]^{-1} \right\}^{-1} = \sum_{h=1}^{H_t} \hat{p}_{th}(x_{th}) \bigg/ \sum_{h=1}^{H_t} \hat{p}_{sh}(x_{th}). \end{aligned} \quad (7)$$

A Fisher-type hedonic index, that treats periods s and t symmetrically, is obtained by taking the geometric mean of Laspeyres and Paasche:⁵

$$\text{F1: } P_{st}^{F1} = \sqrt{P_{st}^{L1} \times P_{st}^{P1}} = \sqrt{\frac{\sum_{h=1}^{H_s} \hat{p}_{th}(x_{sh})}{\sum_{h=1}^{H_s} p_{sh}} \times \frac{\sum_{h=1}^{H_t} p_{th}}{\sum_{h=1}^{H_t} \hat{p}_{sh}(x_{th})}}; \quad (8)$$

⁵Fisher indexes belong to the class of theoretically attractive superlative indexes (see Diewert 1976).

$$F2 : P_{st}^{F2} = \sqrt{P_{st}^{L2} \times P_{st}^{P2}} = \sqrt{\frac{\sum_{h=1}^{H_s} \hat{p}_{th}(x_{sh})}{\sum_{h=1}^{H_s} \hat{p}_{sh}(x_{sh})} \times \frac{\sum_{h=1}^{H_t} \hat{p}_{th}(x_{th})}{\sum_{h=1}^{H_t} \hat{p}_{sh}(x_{th})}}. \quad (9)$$

In the hedonic literature L1, P1 and F1 are referred to as single imputation price indexes, and L2, P2 and F2 as double imputation price indexes (see Hill and Melser 2008). No clear consensus has emerged in the literature as to which approach is better. Single imputation uses less imputations and therefore is preferred by statistical agencies (see de Haan 2004). Double imputation may reduce omitted variables bias (see Hill and Melser 2008). We find that for our data set both the F1 and F2 price indexes and the F1 and F2 rent indexes are almost indistinguishable.

3.2 Hedonic price-rent ratios for individual dwellings

Here we apply the logic of the hedonic imputation method in a new context. Our objective is to compute a matched price-rent ratio for each individual dwelling. We achieve this by first estimating separate price and rent hedonic models. A price for each rented dwelling can then be imputed from the hedonic price model, and a rent for each sold dwelling imputed from the hedonic rent model. In this way a price-rent ratio can be calculated for each rented dwelling and each sold dwelling. A feature of this approach is that the hedonic price and rent models need to be defined on the same set of characteristics. More importantly, we develop extensions of our basic method to account for missing characteristics (i.e., characteristics that are missing for only some dwellings in our data set) and omitted variables (i.e., characteristics that are missing for all dwellings in our data set). These issues are addressed in section 4.⁶

Our starting point is the hedonic price equation, which is assumed to take the following form:

$$y_{Pt} = X_{Pt}\beta_{Pt} + u_{Pt}, \quad (10)$$

where y_{Pt} is the vector of log prices of the dwellings sold in period t , and X_{Pt} is the corresponding matrix of sold dwelling characteristics and u_{Pt} is the random error term with zero

⁶Other papers that use hedonic methods to impute rents for sold dwellings include Arévalo and Ruiz-Castillo (2006), Kurz and Hoffmann (2009), Crone, Nakamura and Voith (2009) and Davis, Lehnert and Martin (2008). Of these only Davis et al. consider the estimation of price-rent ratios, although their focus is narrower than ours.

mean and a constant variance.⁷ Similarly, the hedonic rent equation is as follows:

$$y_{Rt} = X_{Rt}\beta_{Rt} + u_{Rt}, \quad (11)$$

where y_{Rt} is the vector of log rents of the dwellings rented in period t , and X_{Rt} is the corresponding matrix of rented dwelling characteristics. A rent for each dwelling h sold in period t is imputed from (11) as follows:

$$\hat{r}_{th}(x_{Pth}) = \exp\left(\sum_{c=1}^C \hat{\beta}_{Rtc}x_{Pthc}\right), \quad (12)$$

where $c = 1, \dots, C$ indexes the list of characteristics over which the price and rent hedonic models are defined. Similarly, a price for each dwelling j rented in period t is imputed from (10) as follows:⁸

$$\hat{p}_{tj}(x_{Rtj}) = \exp\left(\sum_{c=1}^C \hat{\beta}_{Ptc}x_{Rtjc}\right). \quad (13)$$

We can also use the hedonic rent equation to impute a rent for a dwelling j actually rented in period t :

$$\hat{r}_{tj}(x_{Rtj}) = \exp\left(\sum_{c=1}^C \hat{\beta}_{Rtc}x_{Rtjc}\right), \quad (14)$$

and the hedonic price equation to impute a price for a dwelling h actually sold in period t :

$$\hat{p}_{th}(x_{Pth}) = \exp\left(\sum_{c=1}^C \hat{\beta}_{Ptc}x_{Pthc}\right). \quad (15)$$

The distinction between single and double imputation arises again in the calculation of our hedonic price-rent ratios. A single imputation price-rent ratio $P/R(sold)_{th}^{SI}$ for a dwelling h sold in period t divides the actual price at which dwelling h is sold by its imputed rent in period t obtained from (12):

$$P/R(sold)_{th}^{SI} = \frac{p_{th}}{\hat{r}_{th}(x_{Pth})} = \frac{p_{th}}{\exp\left(\sum_{c=1}^C \hat{\beta}_{Rtc}x_{Pthc}\right)}.$$

A corresponding double imputation price-rent ratio $P/R(sold)_{th}^{DI}$ divides the imputed price

⁷Spatial dependence in our model is captured through the inclusion of postcode dummies. With data on individual dwelling longitudes and latitudes, the spatial dependence could be modeled more rigorously (see for example Bourassa, Cantoni and Hoesli 2007).

⁸Strictly speaking, \hat{r} and \hat{p} are biased estimates of r and p since by exponentiating we are taking a nonlinear transformation of a random variable. Possible corrections have been proposed by Kennedy (1981) and others. From our experience, however, these corrections are small enough that they can be ignored.

for dwelling h obtained from (15) by its imputed rent obtained from (12):

$$P/R(sold)_{th}^{DI} = \frac{\hat{p}_{th}(x_{Pth})}{\hat{r}_{th}(x_{Pth})} = \frac{\exp\left(\sum_{c=1}^C \hat{\beta}_{Ptc} x_{Pthc}\right)}{\exp\left(\sum_{c=1}^C \hat{\beta}_{Rtc} x_{Pthc}\right)}. \quad (16)$$

We can likewise generate two alternative matched price-rent ratios for each dwelling j rented in period t . A single imputation price-rent ratio $P/R(rented)_{tj}^{SI}$ divides the imputed price for dwelling j obtained from (13) by its actual rent:

$$P/R(rented)_{tj}^{SI} = \frac{\hat{p}_{tj}(x_{Ptj})}{r_{tj}} = \frac{\exp\left(\sum_{c=1}^C \hat{\beta}_{Ptc} x_{Rtjc}\right)}{r_{tj}}.$$

Finally, a double imputation price-rent ratio $P/R(rented)_{tj}^{DI}$ divides the imputed price for dwelling j obtained from (13) by its imputed rent obtained from (14):

$$P/R(rented)_{tj}^{DI} = \frac{\hat{p}_{tj}(x_{Rtj})}{\hat{r}_{tj}(x_{Rtj})} = \frac{\exp\left(\sum_{c=1}^C \hat{\beta}_{Ptc} x_{Rtjc}\right)}{\exp\left(\sum_{c=1}^C \hat{\beta}_{Rtc} x_{Rtjc}\right)}. \quad (17)$$

The choice between single and double imputation methods does not affect the general thrust of our results in section 5. The subsequent analysis focuses on the double imputation price-rent ratios.

3.3 Median matched price-rent ratios

Let $Med[P/R(sold)^{DI}]$ denote the median price-rent ratio derived from the double-imputation price-rent distribution defined on the dwellings actually sold, while $Med[P/R(rented)^{DI}]$ denotes the corresponding median price-rent ratio defined on the dwellings actually rented. An overall median is obtained by averaging these two population specific medians as follows:

$$Med[P/R^{DI}] = \sqrt{Med[P/R(sold)^{DI}] \times Med[P/R(rented)^{DI}]}. \quad (18)$$

An alternative approach is to first pool the price-rent distributions defined on sold and rented dwellings and then calculate the median.

$$Med[P/R_{pooled}^{DI}] = Med[P/R(sold)^{DI}, P/R(rented)^{DI}]$$

Intuitively, we prefer the former approach (i.e. averaging rather than pooling) in (18) since it gives equal weight to the price and rent data sets. Empirically we find that the averaged and pooled medians are very close.⁹

⁹As robustness checks, alternative ways of obtaining the estimates of price-rent ratios are considered. The

4 Data Sets and Empirical Strategy

4.1 The hedonic price and rental data sets

The data sets used in this paper are for Australia's largest city, Sydney, over the period 2001 to 2009. These are assembled from three sources. The data pertain to separate houses, where each house is built on its own piece of land. The data set on actual transaction prices is obtained from Australian Property Monitors (APM) and consists of a total of 395,110 observations over the 2001 to 2009 period.¹⁰ The characteristics included in the data set are the transaction price, exact date of sale, land area, number of bedrooms, number of bathrooms, exact address and a postcode identifier. The rental data set is obtained by combining rental data from the New South Wales (NSW) Department of Housing (of which we have 331,877 observations) with data from APM (of which we have 89,495 observations that are not also in the NSW Housing data set). In total, therefore, we have 421,372 rental observations.¹¹

A problem with the data sets is that there are many observations for which one or more characteristics are missing, even after filling in some missing values through the matching of addresses across the three data sets. In particular, all the characteristics are available for 61.67 percent of the price data and for 45.60 percent of the rental data (see Table 1). For the remainder, at least one of the three characteristics of land area, number of bedrooms and number of bathrooms is missing.¹² We explain in section 4.2 how we deal with this problem.

results are shown in Appendix B.

¹⁰APM provides real estate related research service and data for the Australian market. See <http://apm.com.au> in order obtain access to their data sets.

¹¹While the recorded rents in the NSW Housing data refer to new rental contracts, the rents in the APM data refer to rents as advertised in the media. However, we find that there is virtually no difference between the actual and advertised rents when we test their mean difference on the houses which appeared in both data sets in a given quarter.

¹²It seems reasonable to assume that the missing data are randomly missing in the sense that a particular missing value is not related to the value itself. The original sources are government agencies (except for the APM rental data). The physical characteristics information are not important for these agencies and therefore can go missing both at the submission and data entry stage. APM, however, has supplemented the data with characteristic information obtained from other sources (such as newspapers, online housing databases and real estate offices). While this process of supplementing the existing databases could in theory cause the missing characteristics to no longer be random, there is no particular reason to expect such an occurrence.

Insert Table 1 Here

The data sets are expected to provide a comprehensive picture of the purchase and rental markets in Sydney. It is mandatory for the parties to inform the State Valuer-General in the case of any change in the ownership of a property. The Rental Bond Authority obtains the information on new rental contract when the renter deposits the amount of bond with the agency. The authority does not charge any party for their service. While it is not mandatory, most new contracts are recorded with the Rental Bond Authority. Many of the contracts not lodged with the Authority are captured in the APM rental data.

Before proceeding with the estimation of our hedonic models, we removed some extreme observations. We had to undertake some further deletions in order to implement our hedonic approach since it requires that both price and rent models are specified on the same set of characteristics. In total, the deletion of extreme observations and the deletions due to the matching requirement led to the exclusion of 10.6 percent of observations from the total number of price and rental observations, leaving us with 371,604 observations in the price data and 358,381 observations in the rental data (see Table 1 for detailed descriptions of the data sets).

Our expectation is that owner-occupied (and hence sold) dwellings on average are of better quality than rented dwellings.¹³ This hypothesis is supported by the figures in Tables 1 and 2. From Table 1 it can be seen that the mean number of bedrooms and bathrooms and mean land area are all higher for sold dwellings than for rental dwellings. Table 2 compares the bedroom, bathroom, land area and locational distributions of the price and rental data. Of particular interest in Table 2 are the locational distributions. These were constructed by ranking the postcodes from cheapest to most expensive in terms of their median prices and median rents, and then allocating the postcodes to decile groups (i.e., the first decile is the cheapest and the tenth is the most expensive). From Table 2, it is clear that the rented dwellings are concentrated relatively more in the cheaper postcodes.

Insert Table 2 Here

¹³After a house is sold it can be either occupied by the new owner or rented. ABS (2010) reports that the home-ownership rate in Australia remained stable at around 70 percent over the period 1971-2006. This indicates that 70 percent of the houses sold in each year can be expected to be occupied by the new owner. The home-ownership rate in Australia is similar to that of other countries including Canada, New Zealand, the European Union (EU) and the US (see AFTF 2007, and Eurostat 2011).

While these results support the hypothesis that sold dwellings are of better quality than rented dwellings, the quality differences are not that large. When imputing prices for rented dwellings from the price equation and rents for sold dwellings from the rent equation, the mean values of the characteristics corresponding to the predicted dwellings are quite close to the mean values of the characteristics that enter in the corresponding hedonic equations (see Table 1). These factors combined with our large sample size indicate that our approach of imputing prices for rented dwellings and rents for sold dwellings is viable.

4.2 Imputing prices and rents for dwellings with missing characteristics

Dwellings with missing characteristics are a common problem in housing data sets.¹⁴ Instead of deleting these observations, we develop an alternative way of dealing with this issue that may be also applicable in other contexts (such as when estimating price and rent indexes and equivalent rent for owner-occupied houses) and to other data sets (such as electronics data used to construct quality-adjusted price indexes).

Our solution entails estimating a number of different versions of our basic hedonic price and rent equations. This allows the price and rent for each dwelling to be imputed from a hedonic equation that is tailored to its particular mix of available characteristics.

More specifically, focusing on the the case of the hedonic price equation, we estimate the following eight hedonic models (HM1,...,HM8) for each year in our data set:

(HM1): $\ln price = f(quarter\ dummy, land\ area, squared\ land\ area, num\ bedrooms, num\ bathrooms, postcode, land\ area \ \& \ bedroom\ inter., land\ area \ \& \ bathroom\ inter.)$

(HM2): $\ln price = f(quarter\ dummy, num\ bedrooms, num\ bathrooms, postcode)$

(HM3): $\ln price = f(quarter\ dummy, land\ area, squared\ land\ area, num\ bathrooms, postcode, land\ area \ \& \ bathroom\ inter.)$

(HM4): $\ln price = f(quarter\ dummy, land\ area, squared\ land\ area, num\ bedrooms, postcode, land\ area \ \& \ bedroom\ inter.)$

(HM5): $\ln price = f(quarter\ dummy, num\ bathrooms, postcode)$

¹⁴For example, Crone, Nakamura and Voith (2009) mention that they experience this problem with the American Housing Survey (AHS) data set that they use.

(HM6): $\ln price = f(quarter\ dummy, num\ bedrooms, postcode)$

(HM7): $\ln price = f(quarter\ dummy, land\ area, postcode)$

(HM8): $\ln price = f(quarter\ dummy, postcode)$

Each of these eight models is estimated using all the available dwellings that have at least these characteristics. For example, a dwelling for which land area, number of bedrooms and number of bathrooms are all available is included in all eight regressions. A dwelling that is missing the land area is included only in HM2, HM5, HM6, and HM8. A dwelling that is missing land area and number of bathrooms is included only in HM6 and HM8, etc.

The imputed price for each dwelling that is entered into (16) and (17), however, is only taken from the equation that exactly matches its list of available characteristics. This means that a dwelling for which all characteristics are available will have its price imputed from HM1. A dwelling that is missing only land area will have its price imputed from HM2. A dwelling missing land area and number of bathrooms will have its price imputed from HM6, etc.

The imputed rents are obtained in an analogous manner from 8 versions of the hedonic rent equation. If we had only estimated the HM1 model, then the price-rent ratios of a large number of dwellings could not have been calculated. Estimating multiple versions of our hedonic model allows us to calculate the price-rent ratio of every dwelling in the data sets.

4.3 Correcting for omitted variables bias

Omitted variables are a problem in all our hedonic models, even in HM1. The omitted variables may be physical (e.g., the quality of the structure, its energy efficiency, the general ambience, floor space, sunlight, the availability of parking, and the convenience of the floor plan), or locational (e.g., street noise, air quality and the availability of public transport links).¹⁵ Omitted variables bias may also result from nonequivalence between the bedroom and bathroom characteristics in the price and rent data sets. For example, a bathroom in a sold dwelling may on average be of better quality than a bathroom in a rented dwelling.

These two sources of omitted variables bias should reinforce each other since both the

¹⁵The impact of some locational characteristics can sometimes be captured by locational dummies, as long as the geographical zones are sufficiently small. This should be the case for the postcode dummies used here (there are about 213 postcodes in Sydney).

included and omitted characteristics are likely to be of better quality in the price data set than in the rent data set. This implies that our hedonic price-rent ratios failing to fully adjust for quality differences will be biased upward.

Our first step in correcting for omitted variables bias is to obtain reference quality-adjusted price-rent ratios that are free of bias. This can be done by collecting dwellings that are both sold and rented over our sample period. We use a house price index and rent index to extrapolate forwards and backwards prices and rents on the same dwelling in different quarters. For example, suppose dwelling h sells in period s at the price p_{sh} and is rented in period t at the rate r_{th} . An address-matched price-rent ratio for this dwelling in period s can be calculated by extrapolating the rental rate back to period s using a rental index R_{st} as follows:

$$P/R_{sh}^{AM} = \frac{p_{sh} \times R_{st}}{r_{th}}, \quad (19)$$

or by extrapolating the selling price forward to period t using a price index P_{st} as follows:¹⁶

$$P/R_{th}^{AM} = \frac{p_{sh} \times P_{st}}{r_{th}}. \quad (20)$$

The price and rent indexes in (19) and (20) are calculated using Calhoun’s (1996) version of the repeat-rent and repeat-sales method.¹⁷

We now pool all the price-rent ratios derived using (19) and (20), and take the median for each period t :

$$AMm(AMs_t) = Med_{h=1, \dots, H_t}[P/R_{th}^{AM}], \quad (21)$$

where $h = 1, \dots, H_t$ indexes all the address-matched price-rent ratios in period t in our data set. The notation AMs_t in (21) stands for “address-matched sample”, while AMm stands for “address-matched model”. Since it is constructed from actual prices and rents on the same

¹⁶For dwellings with multiple prices and rents in our sample, we select the chronologically closest price and rent observations to construct our address-matched price-rent ratio. For dwellings that sell and rent in the same period, we count these price-rent ratios twice. Hence we have exactly two address-matched price-rent ratios for each dwelling that both sold and rented.

¹⁷Calhoun’s method corrects for heteroscedasticity by giving greater weight to repeats that are chronologically closer together. We prefer the repeat rents/sales method over a hedonic method in this context since the former should be free of omitted variables bias. As a robustness check though we also estimate address-matched price-rent ratios using the price and rent indexes obtained from the double imputation hedonic (F2) method. The two approaches generate similar price-rent ratios.

dwellings, $AMm(AMs_t)$ should by construction be free of omitted variables bias.¹⁸

With our methodology in place for constructing quality-adjusted price-rent ratios that are free of omitted variables bias, we can now compute bias correction factors for models $HM1, \dots, HM8$. We consider first the bias of our $HM8$ model, which is the only one of our hedonic models that can be calculated over the full data set. We calculate this as follows:

$$\lambda_{t, HM8} = \frac{HM8m(AMs_t)}{AMm(AMs_t)}, \quad (22)$$

where $HM8m(AMs_t)$ denotes the median price-rent ratio obtained from (18) using the hedonic model $HM8$ applied to the address-matched sample (AMs) in period t . More precisely, we estimate the $HM8$ model over the full data set and then pick out the imputed price-rent ratios for dwellings in the address-matched sample (AMs). The median is then calculated only over the imputed price-rent ratios in the address-matched sample. The median in the denominator of (22) [i.e., $AMm(AMs_t)$] is obtained from (21).

It should be noted that both medians $HM8m(AMs_t)$ and $AMm(AMs_t)$ are calculated over the same address-matched samples. $HM8m(AMs_t)$ and $AMm(AMs_t)$ separately may suffer from sample selection bias. However, any such biases should be largely offsetting in the adjustment factor specified in (22).

Any systematic deviation of $\lambda_{t, HM8}$ from 1 can hence be attributed to omitted variables bias in the $HM8m(AMs_t)$ median price-rent ratio. In our empirical results we find in every year that $\lambda_{t, HM8} > 1$, indicating that omitted variables bias is causing the price-rent ratios obtained from the $HM8$ model to be systematically too high.

The omitted variables bias for each of our other models HMj (where $j = 1, \dots, 7$) relative to $HM8$ is calculated as follows:

$$\lambda_{t, HMj|HM8} = \frac{HMjm(HMjs_t)}{HM8m(HMjs_t)}. \quad (23)$$

That is, we compare the median price-rent ratio obtained from model HMj estimated over the HMj sample with the median price-rent ratio obtained from $HM8$ estimated over the HMj sample. We use $HM8$ (i.e., the model with the least characteristics) as our reference hedonic

¹⁸Admittedly, when the actual price and rent for a dwelling are not observed in the same period, there is a risk that the quality of the dwelling has changed in the intervening interval (for example it may have been renovated or damaged by flooding). Given that there is little difference in the frequency with which a price observation chronologically precedes a rent observation and vice versa in our address-matched sample, any such quality mismatches should tend to cancel each other out.

model since it can be estimated on any subsample of the data set.

Given that the median imputed price-rent ratios $HMjm(HMjs_t)$ and $HM8m(HMjs_t)$ in (23) are calculated over the same sample of dwellings (i.e., the HMj sample), any systematic deviation of $\lambda_{t, HMj|HM8}$ from 1 can be attributed to missing characteristics in the HMj model. While both $HMjm(HMjs_t)$ and $HM8m(HMjs_t)$ will be subject to bias, our expectation is that the bias will be bigger for $HM8m(HMjs_t)$ than for $HMjm(HMjs_t)$ (for $j = 1, \dots, 7$). This is because the other models include more characteristics than HM8. Given our hypothesis that sold dwelling perform better than rental dwellings on these characteristics, it follows that $\lambda_{t, HMj|HM8}$ should be systematically less than 1. Our empirical results confirm this finding.

Our estimate of the overall omitted variables bias of HMj is then given by:

$$\lambda_{t, HMj} = \lambda_{t, HM8} \times \lambda_{t, HMj|HM8}. \quad (24)$$

That is, first we calculate the omitted variables bias of HM8 (i.e., $\lambda_{t, HM8}$), and then we calculate the bias of model HMj relative to that of HM8 (i.e., $\lambda_{t, HMj|HM8}$). The overall omitted variables bias of model HMj is then obtained by multiplying $\lambda_{t, HM8}$ by $\lambda_{t, HMj|HM8}$.

Our expectation is that $\lambda_{t, HMj} < \lambda_{t, HM8}$ for $j = 1, \dots, 7$ since as already noted each of these other models has less omitted variables. Applying the same logic we should also expect that:

$$\begin{aligned} \lambda_{t, HM1} < \lambda_{t, HM2} < \lambda_{t, HM5} < \lambda_{t, HM8}; & \quad \lambda_{t, HM1} < \lambda_{t, HM2} < \lambda_{t, HM6} < \lambda_{t, HM8}; \\ \lambda_{t, HM1} < \lambda_{t, HM3} < \lambda_{t, HM5} < \lambda_{t, HM8}; & \quad \lambda_{t, HM1} < \lambda_{t, HM3} < \lambda_{t, HM7} < \lambda_{t, HM8}; \\ \lambda_{t, HM1} < \lambda_{t, HM4} < \lambda_{t, HM6} < \lambda_{t, HM8}; & \quad \lambda_{t, HM1} < \lambda_{t, HM4} < \lambda_{t, HM7} < \lambda_{t, HM8}. \end{aligned} \quad (25)$$

For example, taking the first of these six inequalities, we have that HM2 is obtained by deleting land area from HM1. HM5 is then obtained from HM2 by deleting number of bedrooms. Finally, HM8 is obtained by deleting number of bathrooms.¹⁹

We therefore adjust the price-rent ratio of a dwelling h sold in period t with the HMj mix of characteristics for omitted variables bias by dividing it by $\lambda_{t, HMj}$ as follows:

$$P/R(sold)_{th, HMj}^{adj} = \frac{P/R(sold)_{th, HMj}}{\lambda_{t, HMj}} = \frac{P/R(sold)_{th, HMj}}{\lambda_{t, HMj|HM8} \times \lambda_{t, HM8}}$$

¹⁹In Appendix A, we show how the assumption that sold dwellings are better than rented dwellings in terms of omitted characteristics gets reflected in the hedonic price and rent regressions and how this omitted quality difference affects the imputed price-rent ratio at the level of individual dwellings.

$$= P/R(sold)_{th, HMj} \times \left(\frac{AMm(AMs_t)}{HM8m(AMs_t)} \right) \times \left(\frac{HM8m(HMjs_t)}{HMjm(HMjs_t)} \right). \quad (26)$$

Similarly, a dwelling j with the HMj mix of characteristics rented in period t is adjusted for omitted variables bias as follows:

$$\begin{aligned} P/R(rented)_{tj, HMj}^{adj} &= \frac{P/R(rented)_{tj, HMj}}{\lambda_{t, HMj}} = \frac{P/R(rented)_{tj, HMj}}{\lambda_{t, HMj|HM8} \times \lambda_{t, HM8}} \\ &= P/R(rented)_{tj, HMj} \times \left(\frac{AMm(AMs_t)}{HM8m(AMs_t)} \right) \times \left(\frac{HM8m(HMjs_t)}{HMjm(HMjs_t)} \right). \end{aligned} \quad (27)$$

5 Empirical Results

5.1 The estimated hedonic models

We estimate our eight versions of the price and rent hedonic models, HM1–HM8, separately for each of the 9 years in the data set (altogether 144 regressions are run). Focussing on the HM1 model first, which is our most general model, Table 3 provides the average results of some key statistics for the 9 yearly regressions, separately for the prices and rents. The average adjusted R-squares for the price and rent models are 78.4 and 79.3 percent, respectively. The postcode dummies explain 54.9 and 48.1 percent of the variations in the price and rent regressions, respectively. The next largest contribution is the group of physical characteristics, contributing 9.6 and 12.7 percent to the price and rent variations, respectively. The regression results also show that the percentage of significant coefficients is high, their economic significance is plausible and the directions implied by the estimated coefficients accord with our prior expectations. With some small variations in the exact numbers, these results generally hold separately for each of the 9 yearly regressions. Given this performance, our hedonic approach is expected to control for a large portion of the quality difference between sold and rented houses.

Insert Table 3 Here

With regard to the regression results of the HM2–HM8 models, the explanatory power of these models falls as less characteristics are included (as expected), with the smallest model, HM8, explaining 63.9 and 62.8 per cent of the variation in prices and rents, respectively (see Table 4). Around 94.0 per cent of the signs of the estimated coefficients remain the same as the corresponding coefficients of the HM1 model. The premiums to an additional bedroom

or bathroom or more land area are in most cases in HM2–HM7 higher than those found in the HM1 model. This is expected because the estimated coefficients in the HM2–HM7 models include a positive effect of the omitted characteristics. In summary, we find the performance of the HM2–HM8 models is stable across years and is as expected in relation to the HM1 model.

Insert Table 4 Here

5.2 Adjustments for omitted variables bias in our hedonic models

Our distributions of quality-adjusted price-rent ratios, from which medians can then be calculated, are obtained by bringing together the price-rent ratios from our 8 models (HM1, HM2, . . . , HM8). As is explained in section 4.3, a different omitted variables adjustment is made to the imputed price-rent ratios of each model, prior to their pooling into a single data set.

A point of reference is provided by address-matched price-rent ratios, which directly control for quality differences. We have 42,153 dwellings in our data set for which we observe both prices and rents. We have a total of 49,388 selling prices for these dwellings (13.3 percent of the sold data) and 71,566 rents (20.0 percent of the rented data), respectively.²⁰ As shown in (19) and (20), the matching of time periods is attained by extrapolating the prices and rents over time (both backwards and forwards) using price and rent indexes.²¹ The number of houses which are sold and rented more than once within the sample period are 38,612 and 81,017, respectively (corresponding to 81,568 price and 217,575 rent observations).

Table 5-column 2 provides estimates of the omitted variables bias of the price-rent ratios derived from the HM8 model [i.e., $\lambda_{t, HM8}$ derived from (22)]. Conforming to our expectations, we find that for every year $\lambda_{t, HM8} > 1$. The average $\lambda_{t, HM8}$ for 9 years is 1.115, implying that HM8 models fail to fully adjust for the quality difference between the sold and rented dwellings and, as a result, the price-rent ratios obtained from the HM8 model are on average 11.5 percent higher than those obtained from the address-matched model. Table 5 also provides estimates

²⁰The number of observations is greater than the number of dwellings because of repeat-sales and repeat-rents. Only around 1500 of these matched houses were sold and rented in the same quarter.

²¹The average time span over which prices and rents are extrapolated is 2 and a quarter years, with 90 percent of the extrapolation done for less than 6 years (the larger the time span the less reliable is the extrapolation). Harding, Rosenthal and Sirmans (2007) report that the median time between two sales was 5 years for US data.

(see columns 3-9) of $\lambda_{t, HMj|HM8}$ in (23) for $j = 1, \dots, 7$. Conforming to our expectations, these estimates are less than 1 (with only a few exceptions for individual years). This provides strong support for our hypothesis that sold dwellings perform better than the rented dwellings on the omitted variables. A model with more explanatory variables has less omitted variables and hence on average lower price-rent ratios.

Insert Table 5 Here

The overall omitted variables bias $\lambda_{t, HMj}$ of model HMj is obtained by multiplying $\lambda_{t, HM8}$ by $\lambda_{t, HMj|HM8}$, as shown in (24). The estimates of $\lambda_{t, HMj}$ are broadly consistent with the inequalities in (25). While there are some slight inconsistencies for individual years, the average results for each model correspond exactly with (25).

5.3 Quality-adjustment bias in price-rent ratios

Three sets of median price-rent ratios for each year in our data set are presented in Table 6. The first series gives the raw quality-unadjusted price-rent ratios. The second series (quality-adjusted 1) gives the price-rent ratios adjusted for characteristics in our data set derived from (18). The third series (quality-adjusted 2) also accounts for missing characteristics in hedonic models HM2-HM8 and omitted variables. These adjustments are made with reference to the subset of dwellings in our data set that both sell and rent.

The raw quality-unadjusted price-rent ratios are on average 18.4 percent larger than their quality-adjusted counterparts (quality-adjusted 2). From Table 6 it can be seen that slightly under half of this (i.e., 8.7 percent) is attributable to differences in the observable characteristics of sold and rented dwellings. The remainder, 9.7 percent, can be attributed to a combination of missing and omitted characteristics. The relative contributions of each of these can be calculated from the adjustment factor of HM1 in Table 5. On average this adjustment factor is $1.115 \times 0.952 = 1.062$, or 6.2 percent, all of which is omitted variables bias since HM1 is the full hedonic model with no missing characteristics. It follows therefore that the remaining 3.5 percent is attributable to missing characteristics.

The average quality difference between owner-occupied and rented dwellings will be even larger than that between sold and rented dwellings, since some fraction of sold dwellings are subsequently rented. Suppose 70 percent of the dwellings sold in our data set are owner-occupied and 30 percent are rented (as is the case on average in Australia). An estimate of

the average quality difference between owner-occupied and rented dwellings is 26.3 percent – $(118.4 - 0.3 \times 100)/0.7 = 126.3$.

Insert Table 6 Here

It is noticeable that the magnitude of the quality bias falls towards the end of our sample. This pattern can be seen clearly in Table 6 and Figure 1 (as the three curves converge in 2008 and 2009). This seems to be due to a fall in the average quality of dwellings sold after the boom ends in 2004, perhaps due to an increase in the number of distressed sales. Focusing on location as a measure of quality, we find that relatively more houses were sold in cheaper postcodes in the latter part of our sample than in the earlier part. More specifically, over the 2001-5 period, 52.9 percent of sold houses were in the cheapest 4 postcode deciles and 30.8 percent were in the most expensive 4 postcode deciles, whereas over the 2006-9 period 58.4 percent of sold houses were in the cheapest 4 deciles and 25.7 percent in the most expensive 4 deciles. In 2009 this postcode distribution shifted even further to 60.7 and 23.1 percent, respectively. By contrast, the locational distribution of rental dwellings remained relatively stable at around 58.0 percent in the cheapest 4 deciles and 27.0 percent in the most expensive 4 deciles over the entire sample period.²²

Insert Figure 1 Here

5.4 Movement of ratios of price and rent indexes

Figure 2 shows the price and rent indexes obtained from the median, hedonic and repeats methodologies for the whole, lower and upper end of the market. The lower and upper end are defined here as the bottom and top 4 deciles of postcodes, respectively, where these postcodes are ordered from the cheapest to the most expensive, separately for each year.

Insert Figure 2 Here

The three methodologies reveal some common themes. It can be seen that prices rose faster at the lower end during the boom (ending in 2004), after which they fell again. By contrast, at the upper end prices continued to rise after 2004. The movements of rents at the

²²Each yearly regression includes quarterly dummies allowing us to obtain the quality-adjusted price-rent ratios and the estimates of quality bias at quarterly frequencies. Our findings remain the same between the quarterly and yearly results. Therefore, for the purpose of saving space, we produce only the yearly results here.

upper and lower end are more synchronized. Both rose throughout the sample, although at a faster rate at the lower end of the market.

Figure 2 highlights the potential distortions that can arise from using price and rent indexes to measure changes in the price-rent ratio, as is done frequently in the literature (see the discussion in the Introduction). For example, dividing a hedonic price index by a repeat-rent index, or a hedonic price index defined on the upper end of the market by a hedonic rent index defined on the lower end, may generate a distorted price-rent ratio series.

Figure 2 also sheds light on the convergence/divergence trend of the cross-section price-rent ratios. The variance of the log of the price-rent ratio in each quarter is graphed in Figure 3.²³ The dispersion of the price-rent ratios is U-shaped, with the minimum dispersion being observed in 2004. This corresponds to the peak of the boom in Sydney house prices. The U-shape in Figure 3 (i.e., σ convergence followed by later divergence) is explained by the 1993-2004 housing boom in Sydney which started at the high end of the market (triggered by strong income growth at the high end and the scarcity of dwellings in prime locations) and then gradually rippled down to the low end. Towards the end of the boom, the prices of lower quality dwellings rose faster than those at the high end thus causing price convergence. Price rises at the low end however were probably driven more by momentum than genuine scarcity. Also, buyers at the low end tended to have higher loan-to-value ratios. Hence when the boom ended prices fell at the low end, triggered partly by distressed sales, thus generating the subsequent price divergence.²⁴ Meanwhile, the standard deviation of rents over this period was relatively stable. Combining these strands, it follows that price-rent ratios, like prices, first converged and then diverged, generating the U-shaped curve in Figure 3.

Insert Figure 3 Here

²³The variance of the log of the price-rent ratio is scaled up by a factor of 10 to fit in the Figure with the variances of price and rent.

²⁴This perhaps explains why, as is discussed in the previous section, the proportion of sales from cheaper postcodes rose after the boom ended.

6 Detection of Departures from Equilibrium

6.1 Equilibrium versus actual price-rent ratios

The equilibrium price-rent ratio equals the reciprocal of the per dollar user cost u_t , as can be seen by rearranging (1). To calculate u_t , we know from (2) that estimates are needed of the interest rate (r_t), the property tax rate (ω_t), the depreciation rate (δ_t), the risk premium (γ_t), and the expected capital gain (g_t). Here we calculate these parameters as follows.

r is the 10-year interest rate on Australian government bonds (Source: Reserve Bank of Australia). The bond rate remained reasonably stable over the 2001-9 period, ranging between a minimum value of 5.0 percent in 2009 and a maximum value of 6.0 percent in 2007.²⁵

$\omega = 1.0$ percent. This is an estimate for an average land tax over the 2001-2009 period. (Source: Office of State Revenue, New South Wales, Australia)

$\delta = 2.5$ percent. This is the gross depreciation rate estimated by Harding, Rosenthal and Sirmans (2007) using American Housing Survey data over the period 1983 to 2001. To our best knowledge, Harding et al. provide the only micro-data based estimates of the *gross* depreciation rate for housing.

$\gamma = 2.0$ percent. This is the risk premium estimated by Flavin and Yamashita (2002) and used by Himmelberg et al. (2005).

It should be noted that Girouard et al. (2006) fix $\delta + \gamma$ at 4 percent for the 18 OECD countries (including Australia) they studied over the 1990-2004 period. Verbrugge (2008) fixes $\omega + \delta + \gamma$ at 7 percent for the US over the 1980-2004 period. The values of our parameters lie in between these estimates. (The higher the values of these parameters, the more likely it is that the price-rent will be found to be above its equilibrium level.)

g is the expected nominal capital gain which consists of the sum of the expected real

²⁵Alternatively, we could have used the mortgage interest rate. Whether this is appropriate depends on the loan-to-value ratio of purchasers. The relevant interest rate for a purchaser with a 100 percent loan-to-value ratio is the mortgage interest rate r^M , while for a purchaser with a 0 percent loan-to-value ratio it is the risk-free 1-year rate r^{rf} . According to Green and Wachter (2005, Table 2), the average loan-to-value ratio in Australia is 63 percent. Assuming this figure remains constant, we could calculate r as follows: $r^* = 0.37 \times r^{rf} + 0.63 \times r^M$. Interestingly, over our sample, r^* and the 10-year interest rate are quite similar. On average r^* is 0.09 percentage points higher. It follows that the choice between using the 10-year government bond rate and r^* has virtually no impact on our results.

capital gain and expected inflation. The expected real capital gain in year t is assumed to equal the moving average of real capital gain over the preceding x years. We consider three different values of x (i.e., 10, 20 and 30 years). More precisely, the expected real capital gain in year t is calculated as follows:

$$\text{Expected real capital gain}_t = \left(\frac{EHPI_t/CPI_t}{EHPI_{t-x}/CPI_{t-x}} \right)^{1/x}.$$

Here $EHPI_t$ is the level of the Established House Price Index and CPI_t is the level of the consumer price index for Sydney in year t . Both the EHPI and CPI are computed by the Australian Bureau of Statistics (ABS).²⁶ For example, for $x=20$ years, the expected annual real capital gain ranges from a peak of 4.8 percent in 2004 to a low of 1.8 percent in 2009 (see Table 7).

The expected rate of inflation is assumed to be 3 percent. This is very close to the average rate of inflation over the 2001-9 period which equalled 3.07 percent. It is also the upper bound on the Reserve Bank of Australia's inflation target (which is 2-3 percent).

Insert Table 7 Here

Inserting these values into (2) yields the values shown in Table 7 for the equilibrium price-rent ratio $1/u_t$ each year. Table 7 shows that the assumed time horizon of past performance over which expected capital gains are calculated plays a pivotal role. The extreme volatility of per dollar user cost when expected capital gains are extrapolated from past performance over short time horizons has been noted previously by Verbrugge (2008) and Diewert (2009). Diewert, citing evidence on the length of housing booms and busts from Girouard et al. (2006), argues that a longer time horizon (between 10 and 20 years) is more plausible in terms of how market participants form their expectations (see also Bracke 2013).

In the case of Sydney, even 20 years does not seem to be enough, since the strong performance of the housing market in the years leading up to 2004 pushes up the equilibrium price-rent ratio even more than the actual price-rent ratio. We therefore favor extrapolating expected real capital gains over 30 years.²⁷

²⁶The Established House Price Index (EHPI) is computed using the stratified-median approach, which may fail to fully adjust for quality changes over time. Given the EHPI is probably the most widely followed house price index for Sydney, it nevertheless is a useful benchmark for describing expectations of capital gains. The EHPI only goes back to 1986. To obtain prices back to 1981 or 1971 (for the cases where $x=20$ or 30), the EHPI was spliced together with an index calculated by Abelson and Chung (2005).

²⁷When the time horizon is 30 years, the equilibrium price-rent ratio ranges between 17.6 and 22.4. Over

In this case, as can be seen in Table 7, we can conclude that while the price-rent ratio was above its equilibrium level from 2001 to 2008 (by nearly 50 percent in 2004), this was no longer true in 2009. The market has gone through a gradual correction process since 2004. This can be attributed to the combination of stable or falling prices since 2004 accompanied by a steady rise in rents leading to a gradual fall in the price-rent ratio (see Figure 2) and a fall in the 10-year interest rate (see Table 7).

Over our whole sample, the quality-adjusted price-rent ratio is on average 4.3 percentage points above its equilibrium level. This difference rises to 8.3 percentage points in the absence of quality adjustment. Hence quality adjustment essentially halves the magnitude of the measured departure from equilibrium.

6.2 Imputing expected capital gains from the user-cost equilibrium condition

Substituting the values for r_t , ω , δ , γ and π_t^e from Table 7 and the quality-adjusted median price-rent ratios from Table 6 into (3), yields the expected real capital gains series shown in Table 7. The implied expected real annual capital gain if the market is in equilibrium exceeds 4 percent from 2002 to 2007, reaching a peak of 4.7 percent in 2004.

By comparison, Gyourko, Mayer and Sinai (2013) find that the average annual real capital gain for the 50 US cities in their sample over the period 1950 to 2000 was 1.7 percent, with the highest result of 3.5 percent being observed for San Francisco. There are a number of similarities between San Francisco and Sydney, ranging from desirable coastal locations and scarcity of land to population growth. In this sense San Francisco is perhaps not a bad benchmark for Sydney.

We can conclude therefore that the price-rent ratio was above its equilibrium level in Sydney at least from 2002 to 2007. By 2009, however, the implied real expected capital gain in equilibrium had fallen to 2.9 percent, which seems entirely plausible.

In the absence of quality adjustment, the implied expected real capital gain in equilibrium is 5 percent or higher in 2002, 2003, 2004 and 2006. The impact of quality bias on the implied

a 20 year horizon it ranges between 17.4 and 32.0, and over a 10 year horizon between 19.2 to 53.0. If the time horizon is reduced to 5 years, then in some years the equilibrium price-rent ratio rises to infinity since the expected capital gain is large enough to make the user cost become negative.

real expected capital gain in equilibrium is measured by G_t in (4). The largest value of G_t (i.e., 1 percentage point) is observed in 2001. On average over our whole sample, the quality-unadjusted implied expected real capital gains are 0.6 percentage points higher than their quality-adjusted counterparts. Again, therefore, failure to quality adjust leads to the conclusion that the departure from equilibrium (particularly from 2002-7) was larger than it actually was.

6.3 Cross-sectional variation of price-rent ratios

Any variation in price-rent ratios across the housing distribution has implications for the detection of departures from equilibrium. Ordering all dwellings sold and rented each year from cheapest to most expensive, we compute a quality-adjusted price-rent ratio for the lower quartile, median and upper quartile sold dwellings and likewise for the lower quartile, median and upper quartile rented dwellings (see Table 8). The quality-adjusted price-rent ratio in Table 8 is lowest for the first quartile, followed by the median, and is highest for the upper quartile. The price-rent ratio for the lower quartile sold dwelling (in terms of its price) is 23.1 while the corresponding price-rent ratio for the upper quartile is 27.17. The results for the lower and upper quartiles from the rental data are similar, i.e., 22.75 and 26.13 respectively. In other words, we observe that the price-rent ratio increases as we move from the lower to the upper end of the market.

Insert Table 8 Here

This trend has also been noted by Heston and Nakamura (2009), Aten, Figueroa and Martin (2011) and Bracke (2014). Heston and Nakamura for example find that the price-rent ratio rises by more than 50 percent as the price of a dwelling increases from \$50,000 to \$500,000. Their study uses survey data on four regions in the US (Alaska, the Caribbean, the Pacific and Washington D.C.) for 1990, where the rent and price of an owner-occupied dwelling is self-estimated by owners.

According to the user cost equilibrium condition, per dollar user cost should equal rental yield (i.e., the reciprocal of the price-rent ratio). If rental yield is lower at the high end this therefore implies that either per dollar user cost must be likewise lower at the high end or that the equilibrium condition does not hold for all segments of the market. We consider three explanations that focus on the former and two that focus on the latter.

Taking the latter first, the owner of an expensive dwelling may wish to rent to someone reliable who will maintain the property properly. The rent is therefore offered at a discount (see Diewert 2009). Bracke (2014) provides evidence that the duration of tenancies tends to be longer at the high-end of the market, which is consistent with this hypothesis. It follows that rental yield will be lower than per dollar user cost at the high end of the market.

Second, households – particularly those with low incomes or wealth such as young households – may be credit constrained (see Ortalo-Magné and Rady 1999, and Chiuri and Jappelli 2003). They may prefer to buy than rent, but cannot get a large enough mortgage to do so. This will cause rental yield to be higher than per dollar user cost at the low end of the market (see Duca, Muellbauer and Murphy 2011). As is explained later, these first two explanations have implications for the measurement of GDP.

Alternatively, user cost may be lower (and hence the equilibrium price-rent ratio higher) at the high end for one of the following reasons. First, the depreciation rate should be lower at the high end of the market. Structures depreciate while land does not, and the value of the land relative to the value of the structure (sometimes referred to as the land leverage) is typically higher at the high end of the market (see Diewert 2009, and Diewert, de Haan and Hendriks 2014). Himmelberg et al. (2005) make a similar argument in the context of comparisons of price-rent ratios across cities. They argue that the high value of land in San Francisco and New York should act to lower the per dollar user cost in these cities.²⁸ Bracke (2014) suggests a different reason why the depreciation rate may be higher at the lower end of the market. This is that the utilization rate is higher at the low-end of the market (i.e., the density of people for a given floor space is higher).

Second, the risk premium may be lower at the high end of the market, thus lowering the per dollar user cost. Some evidence to support this hypothesis is provided in Figure 2(a), where it is shown that prices at the low end of the market rose more at the end of the boom in Sydney and then fell more as boom turned to bust. This is consistent with Peng and Thibodeau's (2013) finding that house price volatility across markets is negatively correlated with median income. Another possible source of variation in cross-section risk is neighborhood

²⁸Bourassa, Hoesli, Scognamiglio and Zhang (2011) find that the level of land leverage differs significantly over the housing cycle. This likewise has important implications for the interpretation of changes in price-rent ratios over time.

externality risk, which may be higher for low-end homeowners (see Hilber 2005).

Finally, the expected capital gain may be higher at the high end of the market (which again acts to lower the per dollar user cost). While this is unlikely to always be the case, the rise in inequality observed in many countries since the 1970s may be supporting such expectations over a sustained period. Bracke (2014) provides some empirical support for this hypothesis for London.

It follows therefore that to obtain a comprehensive picture of the housing market it is necessary to consider the whole price-rent ratio distribution and the way user cost varies in different segments of the market.

7 Some Implications for the Measurement of GDP

Housing services are an important component of GDP. For example, imputed rent of owner-occupied housing accounts for about 8-9 percent of GDP in the US while tenant rent accounts for about 2-3 percent (see Grist 2010). Our findings have highlighted some of the difficulties that can arise when measuring the flow of housing services. In particular, imputing rent for owner-occupied housing by matching characteristics of rented and owner-occupied dwellings could, in the absence of an omitted variables adjustment, cause the service flow from owner-occupied housing to be underestimated. In our case, we find a 13 percent quality difference between rented and owner-occupied dwellings matched on land area, number of bedrooms, number of bathrooms and postcode. Assuming owner-occupied housing's share in GDP is 8.5 percent, this translates into a downward bias in GDP of over 1 percent (i.e., 0.13×0.085). With less exact matching of characteristics the bias will be even larger.

Mismatches between per dollar user cost and rental yield may also have implications for GDP. When they are not equal, it is not clear which out of user cost and rent (actual or imputed) should be used to measure the flow of housing services. According to Diewert, Nakamura and Nakamura (2009), the value of housing services equals the maximum of rent and user cost (i.e., the opportunity cost). In this case, our results imply that the current methodology that equates the service flow to rent may overstate GDP during booms (where the price-rent ratio is typically above its equilibrium level and hence per dollar user cost exceeds the rental yield), and understate GDP during busts (where the reverse is true).

Over and above these variations over the business cycle, housing services may be over-estimated at the high end of the market (where because of a lack of high income renters per dollar user cost may exceed rental yield) and underestimated at the low end (where because of credit constraints rental yield may exceed per dollar user cost).²⁹

8 Conclusion

We have considered three ways of improving the applicability of the user cost approach for detecting departures from equilibrium in the housing market. First, the actual price-rent ratio needs to be quality adjusted. We have shown how this can be done using hedonic methods that impute prices for rented dwellings and impute rents for sold dwellings. Applying this approach to a data set consisting of 730,000 price and rent observations for Sydney, Australia over the period 2001-9, we find that quality adjustment reduces the actual price-rent ratio by on average 18 percent. Even so, we find that price-rent ratios in Sydney were above their equilibrium level (obtained from the user-cost condition) for most of our sample period.

Second, a crucial input into the equilibrium price-rent ratio formula – the expected capital gain – is not directly observed. When it is calculated by extrapolating past performance, we recommend extrapolating over a long time horizon (e.g., 30 years). When the extrapolation time horizon is too short, the equilibrium price-rent ratio is unstable and prone to rise and fall with the actual price-rent ratio, thus undermining the user-cost approach’s ability to detect departures from equilibrium.

Third, we find that the actual price-rent ratio is higher at the high end of the market than at the low end. We conjecture that the same is probably true for the equilibrium price-rent ratio, primarily due to a lower depreciation rate at the high end and households being credit constrained at the low end. Indeed, in the presence of credit constraints, the method for deriving the equilibrium price-rent ratio itself needs modifying since then equilibrium no longer

²⁹The expansion of the subprime market in the US and other countries in the decade leading up to 2008 may have narrowed the gap between rental yield and per dollar user cost at the low end of the market by allowing more low wealth households to switch from renting to owner-occupying. An implication of the Diewert-Nakamura-Nakamura approach is that failure to account for this trend may impart an upward bias to the growth rate of GDP. The direction of the bias should have reversed since 2008 with the contraction of the subprime market.

implies the equality of the cost of owner occupying and renting. More generally, our cross-section analysis indicates that it is not enough to focus simply on medians when assessing the state of the housing market. One market segment may be in equilibrium while others are not. Ideally, therefore, the user-cost equilibrium condition should be applied to multiple segments of the market (e.g., low, middle, and high).

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Appendix

A. The Impact of Omitted Variables Bias on the Hedonic Price-Rent Ratios

Let the following be the data generating process for the price and rent of sold and rented dwellings, respectively:

$$\ln p_{i,t} = \beta_0 + \beta_1 z_{1,i,t} + \beta_2 z_{2,i,t} + \epsilon_{i,t}^P, \quad \forall i = 1, \dots, I \quad (\text{A-1})$$

$$\ln r_{j,t} = \delta_0 + \delta_1 z_{1,j,t} + \delta_2 z_{2,j,t} + \epsilon_{j,t}^R, \quad \forall j = 1, \dots, J \quad (\text{A-2})$$

where $\ln p_{i,t}$ denotes the log of price of house i in period t and $\ln r_{j,t}$ denotes the log of rent of house j in period t . z_1 and z_2 denote the characteristics of houses. $\epsilon_{i,t}^P$ and $\epsilon_{j,t}^R$ are *i.i.d.* error terms with zero mean and constant variance.

In the following, we consider two models for both prices and rents. *Model1* is estimated on an intercept, z_1 and z_2 , *Model2* on an intercept, and z_1 . The price-rent ratio obtained from Model1 is free of omitted variables bias and is used as the benchmark to compare with the price-rent ratios obtained from Model2. Here z_2 takes the role of an omitted characteristic. We assume that z_2 is a desirable characteristic and is positively correlated with z_1 .

Model1: Let $\widehat{\ln p_h}$ and $\widehat{\ln r_h}$ be the imputed price and rent obtained from Model1 (estimated using OLS) of house h sold in period t (the period subscript is dropped for notational simplicity). The probability limit of the imputed price and rent is calculated as follows:

$$\begin{aligned} \text{plim } \widehat{\ln p_h} &= \beta_0 + \beta_1 z_{1,h} + \beta_2 z_{2,h} = \Theta_h^P \\ \text{plim } \widehat{\ln r_h} &= \delta_0 + \delta_1 z_{1,h} + \delta_2 z_{2,h} = \Theta_h^R \end{aligned}$$

Hence, the probability limit of the log price-rent ratio of house h obtained from Model1 is:

$$plim \ln \left(\frac{p}{r} \right)_h = \Theta_h^P - \Theta_h^R \quad (\text{A-3})$$

Model2: Let $\widetilde{\ln p}_h$ and $\widetilde{\ln r}_h$ be the imputed price and rent of house h sold in period t obtained from Model2. The corresponding probability limits are now calculated as follows:

$$\begin{aligned} plim \widetilde{\ln p}_h &= (\beta_0 + \phi_{02}^P \beta_2) + (\beta_1 + \phi_{12}^P \beta_2) z_{1,h} \\ &= \Theta_h^P + \beta_2 (\phi_{02}^P + \phi_{12}^P z_{1,h} - z_{2,h}) \\ plim \widetilde{\ln r}_h &= \Theta_h^R + \delta_2 (\phi_{02}^R + \phi_{12}^R z_{1,h} - z_{2,h}) \end{aligned}$$

where the ϕ coefficients are the probability limits of the coefficients if we had run auxiliary regressions. For example, ϕ_{02}^P is the probability limit of the intercept coefficient if we had regressed z_2 on a constant and z_1 using the data for sold dwellings, and ϕ_{12}^R is the probability limit of the slope coefficient corresponding to z_1 if we had regressed z_2 on a constant and z_1 using the data for rented dwellings. From the above two equations, the probability limit of the log price-rent ratio obtained from Model2 is:

$$\begin{aligned} plim \ln \left(\frac{p}{r} \right)_h &= (\Theta_h^P - \Theta_h^R) + \beta_2 (\phi_{02}^P + \phi_{12}^P z_{1,h} - z_{2,h}) \\ &\quad - \delta_2 (\phi_{02}^R + \phi_{12}^R z_{1,h} - z_{2,h}). \end{aligned} \quad (\text{A-4})$$

The difference in the probability limit of the log price-rent ratio between Model2 and Model1 is obtained by subtracting equation (A-3) from equation (A-4) as follows:

$$\begin{aligned} plim \left[\ln \left(\frac{p}{r} \right)_h - \ln \left(\frac{p}{r} \right)_h \right] &= \beta_2 (\phi_{02}^P + \phi_{12}^P z_{1,h} - z_{2,h}) \\ &\quad - \delta_2 (\phi_{02}^R + \phi_{12}^R z_{1,h} - z_{2,h}). \end{aligned} \quad (\text{A-5})$$

The omitted characteristic, z_2 , can be better in sold dwellings compared to rented dwellings in the following ways: (1) The characteristic z_2 is valued more by owners than by tenants. For example a well-built structure depreciates more slowly and requires less maintenance (both these costs are borne by owners). In this case we would expect $\beta_2 > \delta_2$; (2) The value of z_2 is larger in sold dwellings compared to rented dwellings (for example sold houses are better built), where this gap does not change with the value of z_1 . In this case $\phi_{02}^P > \phi_{02}^R$; (3) The value of z_2 is larger in sold dwellings and the gap rises as the value of z_1 rises, implying $\phi_{12}^P > \phi_{12}^R$. When at least one of these conditions is satisfied, from equation (A-5) it follows that:

$$plim \left[\ln \left(\frac{p}{r} \right)_h - \ln \left(\frac{p}{r} \right)_h \right] > 0.$$

In such cases failure to control for differences in the characteristics between sold and rented dwellings will, in large enough samples, cause the estimated price-rent ratio to be too large.

B. Robustness Checks

Our base (i.e., preferred) model uses double imputation and corrects for missing and omitted characteristics. In order to check for the robustness of our results, we consider a number of alternatives applied to different stages of the model formulation and estimation process. These alternatives are labeled (a) - (h), and their results are shown in Table B-1. Alternative (a) is expected to produce larger price-rent ratios than the base model because it does not correct for omitted variables bias. With regard to the other alternatives, while most produce results similar to those of the base model, the largest difference is seen in alternative (b), which is obtained using the single imputation method and is on average 3.4 percent higher than its corresponding double imputation counterpart. However, this difference does not affect the general thrust of our results in the subsequent sections.

Table B-1: Robustness Checks

Year	Median Price-rent Ratios Obtained from Different Methods								
	Base	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
2001	20.48	23.35	21.10	20.76	21.05	21.35	20.69	20.08	20.66
2002	24.29	28.19	25.47	24.55	24.52	24.92	24.53	23.98	24.29
2003	28.59	31.81	29.68	28.82	28.51	28.96	29.04	28.66	28.66
2004	29.78	32.11	30.38	29.90	29.10	29.96	30.05	29.53	29.96
2005	27.09	29.48	28.08	26.98	26.94	27.28	27.43	26.37	27.11
2006	25.44	27.76	26.21	25.36	25.89	25.63	25.51	24.84	25.44
2007	23.60	25.31	24.56	23.61	23.65	23.63	23.41	23.35	23.57
2008	21.40	22.47	22.06	21.54	21.88	21.22	21.27	21.11	21.42
2009	21.62	22.46	22.44	21.76	21.31	21.17	21.51	21.34	21.64
Average	24.70	27.00	25.55	24.81	24.76	24.90	24.83	24.36	24.75

Notes: Base case is our preferred model that uses double imputation and corrects for missing and omitted characteristics.

(a) Obtained from running HM1 models on observations which have all three physical characteristics, without correcting for omitted variables bias.

(b) Single imputation price-rent ratios— $P/R(sold)_{th}^{SI}$ and $P/R(rented)_{tj}^{SI}$ (see section 3.2)—are estimated, instead of double imputation price-rent ratios.

(c) Hedonic models in equations (10) and (11) are specified on price and rent levels instead of log prices and rents. In this case, equation (16) is replaced by $P/R(sold)_{th}^{DI} = \sum_{c=1}^C \hat{\beta}_{Ptc} x_{Rthc} / \sum_{c=1}^C \hat{\beta}_{Rtc} x_{Pthc}$ and equation (17) by $P/R(rented)_{tj}^{DI} = \sum_{c=1}^C \hat{\beta}_{Ptc} x_{Rtjc} / \sum_{c=1}^C \hat{\beta}_{Rtc} x_{Rtjc}$.

(d) $Med[P/R_{pooled}^{DI}]$ specified in section 3.3 is used.

(e) Double imputation hedonic price and rent indexes are used in order to estimate P_{st} and R_{st} in equations (20) and (19), respectively (the main results use repeat-sales and -rents indexes).

(f) For dwellings with multiple prices and rents, we consider each price-rent pair in order to obtain P/R_t^{AM} (in equation 21) where the main results use only the chronologically closest price and rent observations.

(g) The regressions required to obtain $\lambda_{t,HM8}$ (equation 22) are estimated using the sample HM8s\AMs (instead of using HM8s which includes AMs) and then the price-rent ratios of the AMs dwellings are imputed from the regression results.

(h) HM1s is used as the reference sample. The correction factor ($\lambda_{t,HMj|HM8}$ equation 23) is obtained using the HM1s, i.e. $\lambda_{t,HMj|HM8} = HMjm(HM1s)/HM8m(HM1s)$.

Table 1: Data Description

Statistics	Price Data	Rental Data
No. of observations	371,604	358,381
Period of coverage (years)	9 (2001–09)	9 (2001–09)
Median price or annual rent (AU \$)	495,000.00	18,250.00
Median land area (square meters)	592.00	584.00
Mean land area (square meters)	684.39 (568.54)	640.86 (390.11)
Median no. of bedrooms	3	3
Mean no. of bedrooms	3.31 (0.84)	3.17 (0.82)
Modal no. of bedrooms	3	3
Median no. of bathrooms	2	1
Mean no. of bathrooms	1.68 (0.74)	1.45 (0.64)
Modal no. of bathrooms	1	1
Percentage of observations having the following characteristics:		
Land area, no. of bedrooms and no. of bathrooms	61.67	45.60
No. of bedrooms and no. of bathrooms	62.43	49.33
Land area and no. of bathrooms	61.67	45.60
Land area and no. of bedrooms	73.50	47.97
No. of bathrooms	62.43	49.33
No. of bedrooms	74.46	99.95
Land area	98.43	47.99

Note: The figures in the parentheses are the estimated standard errors.

Table 2: Distributions of Characteristics in the Price and Rental Data (in %)

Characteristics	Data	Counts							Total		
		1	2	3	4	5	6				
Bedrooms	Price	0.62	13.30	49.40	29.22	6.48	0.98	100.00			
	Rent	1.52	15.48	53.22	23.79	5.88	0.11	100.00			
Bathrooms	Price	46.74	40.07	11.61	1.40	0.18	n.a.	100.00			
	Rent	61.74	31.54	6.20	0.48	0.03	n.a.	100.00			
Characteristics	Data	Deciles									
		1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Land Area*	Price	9.10	10.01	9.99	10.95	9.50	9.97	10.55	9.91	10.19	9.83
	Rent	11.93	10.17	10.15	9.61	9.44	10.03	9.39	9.67	9.54	10.07
Postcodes (by Price) [†]	Price	17.62	14.91	11.79	8.30	9.66	8.28	8.97	8.09	7.33	5.04
	Rent	20.40	15.73	12.35	7.16	8.98	8.09	7.97	7.65	7.15	4.52
Postcodes (by Rent) [§]	Price	18.56	15.46	9.31	7.60	10.96	9.40	9.22	7.91	6.46	5.11
	Rent	19.98	16.97	10.07	7.27	10.55	8.58	8.46	7.69	5.92	4.50

*The price and rental data are pooled before dividing them into deciles in terms of land area. Therefore, each decile corresponds to the same land area in both data sets.

[†]Houses are ordered from the cheapest to the most expensive in terms of price.

[§]Houses are ordered from the cheapest to the most expensive in terms of rent.

Table 3: HM1 Regression Results

Statistics		Price Models	Rent Models
No. of observations		25462 (12273)	18157 (16235)
No. of parameters		204 (17)	204 (17)
Adjusted R^2 (%)		78.35 (3.29)	79.33 (1.95)
Location attributes:	Joint contribution (%)	54.9 (2.89)	48.08 (1.94)
	% of significant coefficients	93.26 (2.10)	85.77 (7.53)
Temporal attributes:	Joint contribution (%)	0.32 (0.33)	0.13 (0.11)
	% of significant coefficients	96.30 (11.11)	55.56 (52.70)
Physical attributes:	Joint contribution (%)	9.60 (0.93)	12.74 (3.76)
	% of significant coefficients	84.13 (8.58)	68.25 (8.07)

Note: The numbers are the mean results obtained from the 9 yearly regressions. The numbers in parentheses are the standard deviations of the 9 yearly regressions, indicating how stable or dispersed the statistics are across yearly regressions. The joint contribution is calculated by taking the difference in the adjusted R^2 between the unrestricted and restricted models. The statistical tests are conducted at the 5% significance level.

Table 4: HM2–HM8 Regression Results

Statistics	Data	HM2	HM3	HM4	HM5	HM6	HM7	HM8
Adjusted R^2 (in %)	Price	75.92 (4.55)	76.67 (2.71)	74.90 (3.65)	74.19 (3.86)	72.36 (5.09)	67.06 (2.87)	63.87 (4.26)
	Rent	78.54 (2.15)	74.44 (1.10)	75.18 (1.96)	73.97 (0.95)	72.50 (2.40)	65.85 (1.74)	62.83 (0.95)
% of coefficients having the same sign as HM1	Price	92.29 (5.43)	98.76 (1.17)	99.20 (6.64)	92.26 (5.18)	91.15 (6.06)	97.32 (1.81)	90.33 (4.34)
	Rent	97.44 (2.15)	95.80 (1.49)	97.04 (1.99)	95.52 (1.89)	92.48 (4.35)	91.37 (2.96)	87.16 (3.06)

Notes: (1) The numbers are the mean results obtained from the 9 yearly regressions. The numbers in parentheses are the standard deviations of the 9 yearly regressions. (2) HM1–HM8 regressions include the following physical characteristics and their interaction terms. HM1: bed, bath, land; HM2: bed, bath; HM3: bath, land; HM4: bed, land; HM5: bath; HM6: bed; HM7: land; HM8: no physical characteristics.

Table 5: Omitted Variables Adjustment Factors: $\lambda_{t,HM8}$ and $\lambda_{t,HMj|HM8}$

Year	$\lambda_{t,HM8}$	$\lambda_{t,HMj HM8}, j = 1, \dots, 8$							
		HM1	HM2	HM3	HM4	HM5	HM6	HM7	HM8
2001	1.160	0.943	0.946	0.944	0.981	0.944	0.981	0.989	1.000
2002	1.169	0.948	0.951	0.947	0.980	0.948	0.987	0.993	1.000
2003	1.120	0.943	0.945	0.944	0.977	0.948	0.981	0.993	1.000
2004	1.115	0.939	0.953	0.939	0.975	0.953	0.987	0.992	1.000
2005	1.103	0.962	0.967	0.960	0.987	0.962	0.992	0.999	1.000
2006	1.121	0.960	0.963	0.962	0.987	0.963	0.991	0.995	1.000
2007	1.117	0.954	0.949	0.966	0.975	0.960	0.974	1.002	1.000
2008	1.084	0.956	0.956	0.966	0.973	0.964	0.976	1.005	1.000
2009	1.052	0.965	0.960	0.972	0.980	0.968	0.984	1.004	1.000
Average	1.115	0.952	0.954	0.955	0.979	0.957	0.984	0.997	1.000

Note: Overall adjustment factor: $\lambda_{t,HMj} = \lambda_{t,HM8} \times \lambda_{t,HMj|HM8}$. For example, $\lambda_{2001,HM1} = 1.160 \times 0.943 = 1.094$.

Table 6: Actual and Quality-Adjusted Median Price-Rent Ratios and Quality Bias

Year	Actual	Quality	Quality	Bias 1 (%)	Bias 2 (%)
	Unadjusted	Adjusted 1	Adjusted 2		
2001	25.63	23.20	20.48	10.46	25.15
2002	30.42	28.10	24.29	8.24	25.24
2003	34.72	32.03	28.59	8.38	21.45
2004	35.48	32.33	29.78	9.74	19.13
2005	33.41	29.55	27.09	13.11	23.32
2006	32.06	27.79	25.44	15.36	26.01
2007	27.45	25.13	23.60	9.23	16.33
2008	23.01	22.22	21.40	3.55	7.52
2009	21.85	22.13	21.62	-1.26	1.10
Average	29.34	26.96	24.70	8.70	18.36

Notes: (1) The quality-adjusted 1 series is calculated from (18). It quality adjusts based on the characteristics available in our data set. The quality-adjusted 2 series also accounts for missing characteristics in some of our hedonic models and omitted variables (with the latter adjustments made with reference to the subset of dwellings in our data set that both sell and rent). (2) Bias k = $100 \times [(Actual/Quality-adjusted k) - 1]$.

Table 7: A Comparison of Equilibrium and Actual Price-Rent Ratios

Year	r_t	$g_t - \pi_t^e$			Equilibrium P_t/R_t			Actual P_t/R_t		Implied $g_t - \pi_t^e$	
		$x=10$	$x=20$	$x=30$	$x=10$	$x=20$	$x=30$	Adjust	Unadj.	Adjust	Unadj.
2001	0.056	0.029	0.024	0.024	19.18	17.39	17.64	20.48	25.63	0.032	0.042
2002	0.058	0.045	0.037	0.028	26.11	21.45	18.10	24.29	30.42	0.042	0.050
2003	0.054	0.060	0.047	0.031	52.88	32.02	21.10	28.59	34.72	0.044	0.050
2004	0.056	0.062	0.048	0.033	52.97	30.33	20.92	29.78	35.48	0.047	0.053
2005	0.053	0.059	0.047	0.033	47.57	29.89	21.94	27.09	33.41	0.041	0.048
2006	0.056	0.056	0.043	0.033	38.47	27.06	21.03	25.44	32.06	0.042	0.050
2007	0.060	0.053	0.045	0.036	32.41	25.50	20.29	23.60	27.45	0.043	0.048
2008	0.058	0.048	0.042	0.033	24.66	19.79	20.25	21.40	23.01	0.036	0.039
2009	0.050	0.034	0.018	0.030	26.14	19.47	22.40	21.62	21.85	0.029	0.029
Mean	0.056	0.049	0.038	0.031	35.60	24.77	20.41	24.72	29.34	0.040	0.046

Notes: $\omega_t = 0.01$, $\delta_t = 0.025$, $\gamma_t = 0.02$ and $\pi_t^e = 0.03$ for the entire period.

Table 8: Quality-Adjusted Price-Rent Ratios for Different Market Segments

Houses are ordered from cheapest to most expensive

Year	Price-Rent from Price Data			Price-Rent From Rent Data		
	Lower Quartile	Median	Upper Quartile	Lower Quartile	Median	Upper Quartile
2001	19.28	21.89	22.63	18.49	20.09	21.82
2002	21.17	24.79	26.98	21.72	24.20	25.96
2003	27.39	29.17	31.47	26.27	28.91	30.16
2004	28.61	30.11	30.91	28.35	29.99	30.14
2005	26.52	27.07	27.35	26.14	27.36	28.19
2006	23.88	26.02	29.40	23.67	24.93	25.84
2007	21.57	25.65	25.54	21.24	23.01	25.66
2008	19.70	21.07	24.08	19.47	21.26	23.58
2009	19.82	21.36	26.13	19.36	21.27	23.81
Average	23.10	25.24	27.17	22.75	24.56	26.13

Figure 1: Quality-Adjusted and Unadjusted Price-Rent Ratios

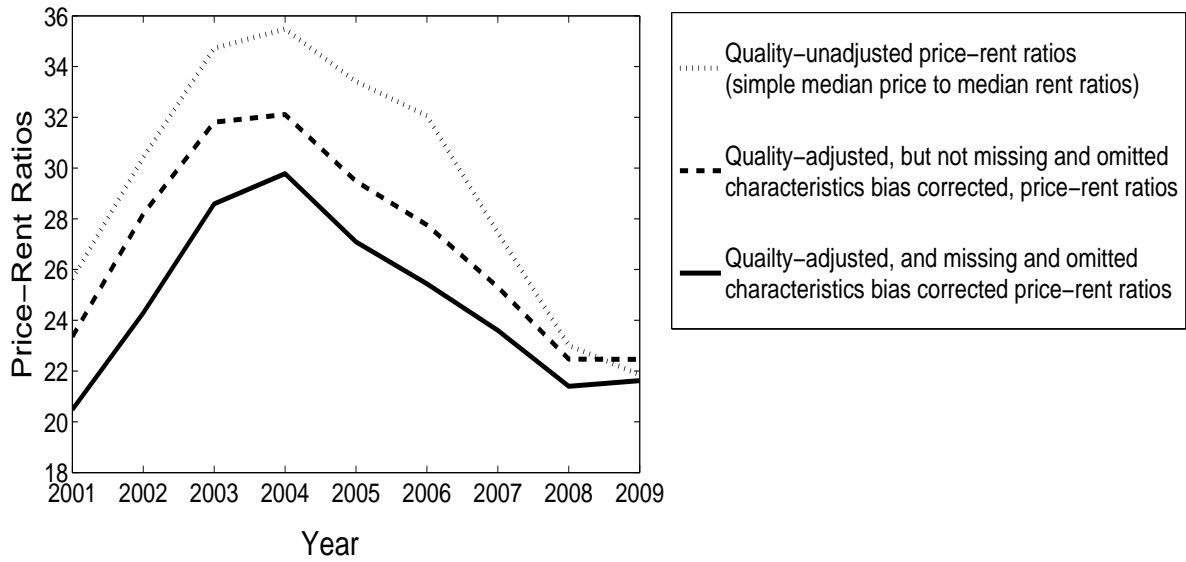


Figure 2: Price and Rent Indexes of Lower and Upper Ends of the Market

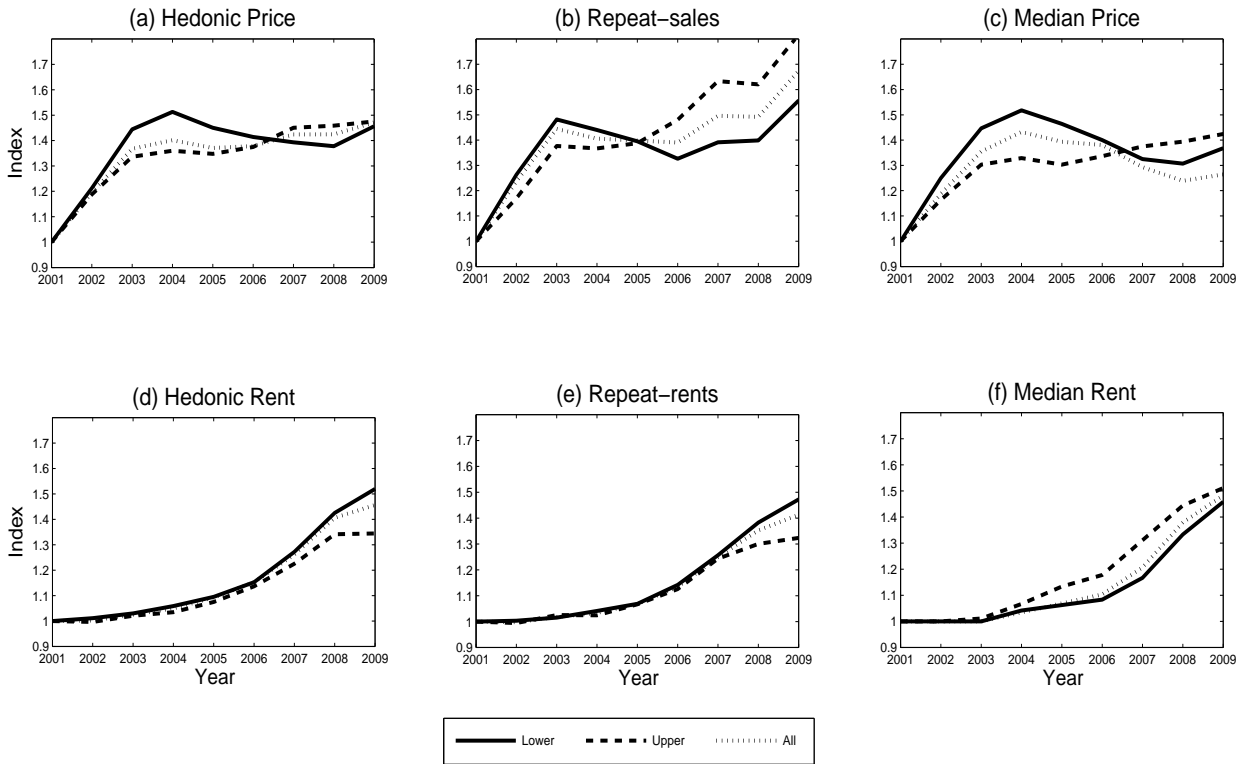


Figure 3: σ -Convergence

