First- and Second-order Subjective Expectations in Strategic Decision-Making: Experimental Evidence

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Abstract

In this paper we study first- and second-order subjective expectations in strategic decision-making. We propose a method to elicit probabilistically not only first- but also second-order expectations and apply the method to a Hide-and-Seek experiment. The proposed method consists of two steps. In the first step, subjects report what they think the most likely value is for their opponent's first-order expectations. In the second step, subjects report the probabilities with which their opponent's first-order expectations will fall within several intervals. The coherence of the elicited second-order expectations is supported by both nonparametric and parametric analysis. We study the relationship between choice and expectations in terms of best-response requirements and the relationship between first- and second-order expectations under two criteria which we define as *belief-consistency* and *behavior-consistency*. Belief-consistency requires an agent to believe that her opponent best responds to first-order expectations. Behavior-consistency instead only requires first- and second-order expectations to prescribe the same best-response behavior. We find that experiment participants are more likely to make choices that are consistent with their first-order expectations than choices consistent with their second-order expectations. Moreover, while consistency between choice and first-order expectations or between choice and second-order expectations occurs often, belief-consistency or behavior-consistency between first- and second-order expectations is rarely achieved. By measuring second-order expectations probabilistically, we can also examine characteristics of the decision process that would otherwise be unobservable, such as the relationship between the uncertainty inherent in second-order expectations and the consistency between second-order expectations and choice behavior. Contrary to what has been conjectured in previous work, we do not find evidence that more uncertain expectations make it less likely for choice and expectations to be consistent with each other.

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1 Introduction

Many strategic situations in business, politics and social life can be modeled by a game in which players try to outguess each other by anticipating correctly each other's actions. In such situations taking a decision does not simply involve choosing the best action given what we think other people will do, but also choosing the best action given what we think other people think we will do.

Consider two subjects, 1 and 2, trying to outguess each other. Suppose that both subjects need to choose between two alternatives, A and B. While subject 1's first-order beliefs are her beliefs about the choice that subject 2 will make, subject 1's second-order beliefs are her beliefs about the beliefs that subject 2 in turn holds about her own choice.[1] Given that subjects are likely to feel uncertain about what the unknown choice of other subjects will be, first-order beliefs are best represented in terms of probabilities. Subject 1 attaches a probability value to each possible choice of subject 2, A and B. Then, second-order beliefs, as they are beliefs about probabilistic first-order beliefs, are in fact beliefs about a probability. For this reason, probabilistic second-order beliefs are not simply a probability value, but a probability distribution.

This paper studies first- and second-order subjective expectations in a strategic decision-making experiment. Our objective is to investigate the feasibility of eliciting both first- and second-order beliefs probabilistically and to examine the relationship between choice and elicited probabilistic first- and second-order beliefs. We design and conduct a Hide-and-Seek experiment, in which both first- and second-order expectations are measured probabilistically.

The Hide-and-Seek game is a simple 2-person game representing a vivid strategic situation. A player, the Hider, has to choose where to hide a prize (a $10 banknote) between two possible locations, labeled A and B, and another player, the Seeker, has to guess where the prize will be hidden. The Seeker wins the prize if she guesses correctly where the prize is hidden; otherwise the Hider wins the prize.[2] The Hide-and-Seek game has a structure similar to a matching pennies game.[3] However, the Hide-and-Seek game is strategically more interesting than the matching pennies game because of the introduction of an asymmetry in framing, which makes one location (location A) a focal point.

The main contribution of this paper is to show that elicitation of probabilistic second-order expectations, along with first-order expectations, is feasible and to propose a methodology to achieve the elicitation. The proposed method consists of two steps. In the first step, subjects report what they think the most likely value is for their opponent's first-order beliefs. In the second step, subjects report the probabilities with which their opponent's first-order beliefs will fall within several intervals. The two steps require subjects to report two different types of information about their subjective second-order beliefs. The first step requires to state a point forecast, while the second step allows for a probabilistic forecast defined by probabilities over several ranges of possible values.

The feasibility of eliciting second-order beliefs probabilistically would be severely undermined were the answer in terms of a one-point forecast and the answer in terms of a probabilistic forecast not coherent. Coherence requires that the 'most likely value' (the one-point forecast) is in fact a measure of

[1] Second-order beliefs are also called, in the psychology literature, self-referential beliefs.

[2] For details, see Section 2 and the instructions reported in Appendix A.

[3] The matching pennies game can be considered the simplest game in which players try to outguess each other. The matching pennies game is played between two players, Player A and Player B. Each player has a penny and must secretly turn the penny to heads or tails. The players then reveal their choices simultaneously. If the pennies match (both heads or both tails), Player A receives one dollar from Player B. If the pennies do not match (one heads and one tails), Player B receives one dollar from Player A.

central tendency for the subjective distribution that is characterized by the probabilities over intervals (the probabilistic forecast). It is reasonable to expect that, if subjects were unable to state their second-order beliefs probabilistically, while they found it quite natural to state them as a one-point forecast, requiring them to submit both a one-point and a probabilistic forecast would likely result in incoherent answers.[4] Evidence from both nonparametric and parametric analysis (presented in Appendix A) suggests that the elicited second-order expectations are coherent.

In terms of choices and outcomes, we find that alternative A is chosen by hiders in 60% of the observations and by seekers in 40% of the observations and that seekers win the game slightly less often than hiders (48% of the time). Inspecting the answers to the question about first-order expectations as well as the answers to the question about the 'most likely value' of one's opponent's beliefs, we find significant differences across observable characteristics of the sample: female participants and participants with a major in the Social Sciences tend to report the central answer of 50% more often than male participants and participants with a major in the Sciences or in the Humanities.

We study the relationship between choice and expectations in terms of best-response requirements. We find that choice and first-order expectations are consistent with each other in 89% of the observations, while choice and second-order expectations are consistent with each other in 75% of the observations. Restricting to those observations for which both first- and second-order beliefs rule out indifference in best-response, the percentages decrease to 81% and 57%. In both cases, the results are in contrast with previous experimental work that uses nonprobabilistic beliefs (thus ignoring the effect of indifference in best-response) and that reports rates of consistency between choice and second-order beliefs higher than those between choice and first-order beliefs (see Bhatt and Camerer (2005)).

We study the relationship between first- and second-order expectations under two criteria which we define as *belief-consistency* and *behavior-consistency*. Belief-consistency requires an agent to believe that her opponent best responds to first-order expectations. Behavior-consistency instead only requires first- and second-order expectations to prescribe the same best-response behavior. Choice, first- and second-order expectations are all simultaneously consistent with each other in 33% of the observations (under the *belief-consistency* criterium) and in 44% of the observations (under the *behavior-consistency* criterium). Restricting to those observations for which both first-order beliefs and second-order beliefs rule out indifference in best-response, the percentages are 17% and 47% under *belief-consistency* and *behavior-consistency*, respectively. Measuring expectations probabilistically also allows investigating a previously unobserved relationship: the relationship between the uncertainty inherent in second-order expectations and the consistency between second-order expectations and choice behavior. Contrary to what has been conjectured in previous work, we do not find evidence that more uncertain second-order beliefs make it less likely for choice and second-order beliefs to be consistent with each other.

The results of this paper provide encouraging evidence in favor of the feasibility of measuring second-order beliefs probabilistically. Moreover, while the proposed elicitation method is implemented within a lab experiment, we believe that its format and wording could prove useful also outside the lab. The feasibility of measuring second-order beliefs probabilistically represents a step forward in understanding the process of thinking that subjects experience when facing a strategic situation, and in turning the game-theoretic concept of higher-order beliefs into an observable variable.

This paper is closely related to the literature on elicitation of probabilistic first-order beliefs.

[4] While surveys are often plagued by nonresponse, the experiment conducted in this study is not subject to nonresponse, since experiment participants are required to complete all forecast tasks. Therefore, we refer to subjects being possibly 'unable' to state second-order beliefs probabilistically, and don't refer to them being possibly 'unwilling'.

Researchers have elicited probabilistic beliefs for over a century and the practice has become common in survey research since the early 1990s. See the review article of Manski (2004). Elicitation of first-order beliefs in experimental economics is much more recent. Nyarko and Schotter (2002) studied a 2x2 normal-form game with a unique mixed-strategy Nash equilibrium and show how the elicitation of first-order probabilistic beliefs can improve the prediction of choice behavior compared to the use of unverifiable proxies for beliefs.[5] Manski (2002) showed how probabilistic beliefs data can enable to overcome the identification problem that arises when choice data alone is used to make inference about decision rules. Since then, elicitation of first-order beliefs in experiments has grown rapidly.

While elicitation of first-order beliefs has received growing attention recently, attempts to elicit second-order beliefs have been limited. Not only have the attempts been limited, but, as far as we are aware, previous research that measures second-order beliefs measures them either in terms of actions or in terms of point-forecasts[6], and not in terms of probability distributions, as has become standard practice for the elicitation of first-order beliefs. To our knowledge, this paper represents the first attempt to measure second-order beliefs probabilistically.

Given the specific experimental setting (i.e., a Hide-and-Seek game with a nonneutral framing), this paper is related to Rubinstein, Tversky and Heller (1996) and Crawford and Iriberri (2007). Crawford and Iriberri (2007) explain the deviations of choice behavior from equilibrium predictions, observed in the experimental data collected by Rubinstein, Tversky and Heller (1996), using a structural nonequilibrium model based on 'level-k" thinking.

The remainder of the paper is organized as follows. Section 2 presents the experimental design and the method used to elicit subjective first- and second-order beliefs probabilistically. Section 3 describes the decision problem and defines the relevant expectations variables and the concepts of *belief-consistency* and *behavior-consistency*. Section 4 presents the main results. Finally, section 5 concludes, suggesting directions for further research.

2 Experimental Procedure

The experiment was conducted in the Computer Laboratory of the Main Library at Northwestern University in Evanston, IL. Participants were undergraduate students from Northwestern University.

[5] A rapidly-growing literature analyzes the relation between choice behavior and beliefs. Rutstrom and Wilcox (2007) and Palfrey and Wang (2009) focus on the effects that the elicitation of beliefs may have on choice behavior, including the possibility of more strategic behavior, lower risk aversion and overconfidence. Fehr, Kübler and Danz (2010) focus on the role that participants' matching mechanism, feedback about previous outcomes and information about opponent's payoff may have on the relation between choice behavior and beliefs, arguing that feedback about previous outcomes is the driving force of learning. Costa Gomes and Weizsacker (2008) study one-shot games with no feedback about outcomes nor opponent's behavior and find that choices are often not best response to first-order beliefs. Their suggested explanation is that subjects neglect the incentives of their opponent more when they choose their action than when they state their predictions about the opponent's behavior. Their results suggest that subjects play the game as if expecting the opponent to play randomly, and state beliefs as if thinking that the opponent will choose her best response to beliefs which are uniform over the player's own decision. Most of this literature has focused on normal-form games. Belief elicitation in extensive-form games is implemented by Dominitz and Hung (2004) and Ziegelmeyer, Bracht, Koessler and Winter (2010) in order to study social learning in information cascade games.

[6] Second-order beliefs are elicited in terms of actions in Bhatt and Camerer (2005) and in terms of point-forecasts in Vanberg (2008) and Bellemare, Sebald and Strobel (2010). The unpublished version which preceded Costa Gomes and Weizsacker (2008) also contained elicitation of second-order beliefs in terms of point-forecasts.

Subjects were recruited using the online recruitment system ORSEE (Greigner (2004)) and the experiment was programmed and conducted with the software Z-Tree (Fischbacher (2007)).

Each experimental session lasted for approximately 30 minutes, including the time for reviewing the instructions, and was identically administered by the same experimenter. When the subjects first arrived at the Computer Lab, they were randomly assigned to one of the 30 computer terminals in the Lab. A welcoming speech was then given, describing the structure and timing of the experiment. Finally, a three-page copy of the instructions was distributed to all participants, who then had 5 minutes to read the instructions and ask questions. Students who wished to ask questions would raise their hand and their questions would be answered privately. Students were allowed to keep a copy of the instructions during the entire session.

The participants play a Hide-and-Seek game. The game is played in pairs: one subject is given the role of Hider and the other subject the role of Seeker. Henceforth, the Hider and Seeker are called agents H and S respectively. The Hider has to hide a prize (a $10 banknote) in one of two locations. The Seeker has to guess where the prize has been hidden. If the Seeker guesses correctly, she wins the prize. Otherwise, the Hider keeps the prize. The two locations are two zones in which a square field has been divided. The two zones have the same area, while they differ in shape and labeling. There is an inner square field, labeled A, and an outer contour-shaped field, labeled B. The instructions in Appendix A contain the figure representing the two zones.

Several designs of Hide-and-Seek games have been studied in situations with nonneutral payoffs and/or framing of locations. In the design used by Rubinstein, Tversky and Heller (1996), the Hider has to choose to hide a prize in one of four identical boxes, lined one next to each other and labeled, from left to right, as box A, B, A, A. In the game studied by Ayton and Falk (1995), "hide a treasure in a $5X5$ table", the Hider has to hide a treasure in one of the table's 25 boxes. The game used in this paper, by having a design with only two alternatives (A and B), simplifies considerably the elicitation of probabilistic expectations. At the same time, as in the design with more than two alternatives, the game preserves a nonneutral framing: the inner region is the focal location, despite the fact that inner and outer regions have the same area.

Six different treatments were implemented. Each treatment differs from the others in the order in which subjects are asked to report their choice, first-order expectations and second-order expectations. Each treatment is assigned randomly to one session. Every other element, except the order of decisions, is identical among treatments. The treatments are:

- treatment C-1-2: choice, 1st-order expectations, 2nd-order expectations
- treatment C-2-1: choice, 2nd-order expectations, 1st-order expectations
- treatment 1-C-2: 1st-order expectations, choice, 2nd-order expectations
- treatment 1-2-C: 1st-order expectations, 2nd-order expectations, choice
- treatment 2-1-C: 2nd-order expectations, 1st-order expectations, choice
- treatment 2-C-1: 2nd-order expectations, choice, 1st-order expectations

At the beginning of each round, subjects were matched randomly into pairs and roles were assigned randomly within each pair. Each participant was informed of the role assigned to him/her for that round: information would appear on the computer screen for the entire duration of the round. At the end of each round, each subject received feedback information consisting of: (i) whether or not he/she won the $10 prize, (i) the total amount of money earned in forecasting his/her opponent's choice and his/her opponent's beliefs about his/her own action. Therefore, from the feedback information it is immediate to infer the opponent's actual choice, but it is not possible to infer the opponent's actual beliefs.

The subjects played for 4 rounds. When the last round ended, the computer randomly drew one of the rounds and the participants were paid according to their performance in that round only. Once the experiment was over, subjects filled in a questionnaire while waiting to be paid. The questionnaire consisted of questions about each participant's gender, age, major, year of graduation, familiarity with the game, and number of classes taken in (i) economics, finance or accounting, (ii) mathematics, and (iii) psychology. The participants were also given the option to leave specific comments about the way they played the game and/or general comments about the experimental session. Subjects were paid individually in a sealed envelope. Payments included $5 for attending the session, plus the amount earned in the experiment itself. On average, subjects earned approximately $13 for their participation ($5 from the show-up fee, $5 from the choice task and $2.93 from the first- and second-order beliefs tasks).

2.1 Elicitation of Probabilistic Expectations

In this section we describe the methods used to elicit first- and second-order expectations.

2.1.1 Elicitation of First-Order Expectations

The wording of the question used to elicit first-order expectations is reported below. Question $1H$ elicits what the Hider believes to be the probability that her opponent will choose A and B. Question $1S$, not shown here, is the analogous question presented to the Seeker.

QUESTION (1H)

What do you think the percent chance is that the Seeker will look for the prize in A? And in B?
Write your answers in the spaces provided below.
You can choose values between 0 and 100.
The values you choose should sum to 100.

Percent chance that the Seeker will look for the prize in A: ...

Percent chance that the Seeker will look for the prize in B: ...

The answers to questions $1H$ and $1S$ are remunerated using a quadratic scoring rule. The reward is paid in dollars. In order to illustrate the rule, consider a subject with the role of Hider who has reported probabilities P_H and $1 - P_H$ as the probabilities with which the Seeker will choose A and B respectively. Then, the Hider's reward according to the quadratic scoring rule will be:

$$S_H(I_A, I_B, P_H) = 2 - \left\{[I_A - P_H]^2 + [I_B - (1 - P_H)]^2\right\} \tag{1}$$

where I_A is an indicator function that takes value 1 if the Seeker chooses A and 0 otherwise, and I_B is an indicator function that takes value 1 if the Seeker chooses B and 0 otherwise.

Therefore, if the Seeker chooses A, then the Hider would earn the highest reward by assigning all the probability weight on A, i.e. $P_H = 1$. If the Hider assigns $P_H < 1$ to alternative A and $1 - P_H > 0$ to alternative B, then she will be penalized for both mistakes: for assigning a probability smaller than 1 to A and for assigning a probability larger than 0 to B. The first mistake will cause a penalty of $[1 - P_H]^2$ and the second mistake will cause a penalty of $[0 - (1 - P_H)]^2$. Both penalties will be subtracted from the maximum possible reward of \$2. The minimum possible reward is \$0.

2.1.2 Elicitation of Second-Order Expectations

The elicitation of second-order expectations is divided into two questions. The two questions elicit information about the beliefs that a subject holds about the beliefs of her opponent. Since second-order beliefs are beliefs about probabilistic first-order beliefs, they are in fact beliefs about a probability. For this reason, probabilistic second-order beliefs are not simply a probability value, but a probability distribution.

In the first question, participants are required to report what they think the 'most likely value' is for the opponent's answer to the first-order beliefs task. This question is meant as an introduction and a convenient way for participants to better understand the second question, which is the crucial part in the elicitation of probabilistic second-order beliefs. In fact, as Section 3 illustrates, the key necessary data regarding second-order beliefs, in order to then analyze their role within the decision-making problem, does not consist of a measure of central tendency (as eliciting the 'most likely value' may suggest) but of a measure of the cumulative distribution function characterizing second-order beliefs.[7] The second question allows to collect the necessary information about such cumulative distribution function. Participants are required to report the probabilities with which the opponent's answer will fall within several different ranges of values. The intervals are: $[0, 5]$, $(5, 20]$, $(20, 50]$, $(50, 80]$, $(80, 95]$ and $(95, 100]$.[8]

The wording of each question used in the experiment is reported below. Questions $2H$ and $3H$ elicit the Hider's second-order beliefs and questions $2S$ and $3S$, not shown here, elicit the Seeker's second-order beliefs.

QUESTION (2H)

You are the Hider.

For sure your opponent wants to find the prize, so he or she must be trying to guess where you will hide it.

We've just asked your opponent to tell us what he or she thinks. The question we asked was: What do you think the percent chance is that the Hider will hide the prize in A?

Your opponent has answered this question. You don't know the answer. How do you think your opponent has answered?

Tell us what you think the most likely value is for the answer given by your opponent.

[7] Therefore, the elicited answers in terms of 'most likely value' will not used in Section 4.

[8] Notice that the previous answer in terms of 'most likely value' does not influence the definition of the intervals in the second question, which are fixed.

I think the most likely value for the Seeker's answer is: ...

QUESTION (3H)

Now tell us something more. Please complete the following sentences.

I think that the percent chance that the Seeker's answer is not larger than 5 is: ...

I think that the percent chance that the Seeker's answer is larger than 5 and not larger than 20 is: ...

I think that the percent chance that the Seeker's answer is larger than 20 and not larger than 50 is: ...

I think that the percent chance that the Seeker's answer is larger than 50 and not larger than 80 is: ...

I think that the percent chance that the Seeker's answer is larger than 80 and not larger than 95 is: ...

I think that the percent chance that the Seeker's answer is larger than 95 is: ...

The answers to questions $2H$ and $2S$ are remunerated using a zero-one scoring rule[9]. The reward is paid in dollars. The zero-one scoring rule rewards an agent if her reported 'most likely value' coincides with the first-order beliefs stated by her opponent. In order to illustrate the rule, consider a subject, with the role of Hider, reporting m as the 'most likely value' for the Seeker's first-order beliefs P_S. Then, the Hider's reward according to the zero-one scoring rule will be:

$$S_H(m, P_S) = \begin{cases} 2 & \text{if } P_S = m \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

The answers to questions $3H$ and $3S$ are remunerated using a quadratic scoring rule. In order to illustrate the rule, consider a subject, with the role of Hider, assigning the probability vector $p = (p_{[0,5]}, p_{(5,20]}, p_{(20,50]}, p_{(50,80]}, p_{(80,95]}, p_{(95,100]})$ to the intervals $[0, 5]$, $(5, 20]$, $(20, 50]$, $(50, 80]$, $(80, 95]$, and $(95, 100]$. Then, the Hider's reward will be:

$$S_H(p, I) = 2 - \left(\sum_{j=1}^{6} (I_{[x_j, y_j]} - p_{[x_j, y_j]})^2 \right) \tag{3}$$

where $I_{[l,r]}$ is an indicator function that takes value 1 if the Seeker has reported as her first-order belief a percentage chance which lies in the interval $[l, r]$ and 0 otherwise, and $p_{[l,r]}$ is the Hider's belief that the first-order belief reported by the Seeker lies in the interval $[l, r]$. The score ranges between \$0 and \$2. The worst possible guess, i.e. assigning 100% chance to an interval while the correct value lies in another interval, yields a payoff of \$0. The best possible guess, i.e. assigning 100% chance to the interval where the correct value lies, yields a payoff of \$2.

[9]It is easy to show that a zero–one scoring rule rewards a probabilistic forecast if the mode of the predictive distribution materializes. We don't intend to stress the linkage between the forecast and the mode, since we don't make any argument that subjects are in fact expressing the mode of their subjective expectations. We chose this scoring rule mainly for the easiness in which it can be understood by subjects.

3 Choices and Expectations

In this section we illustrate the Hider's and Seeker's decision problem and show how choices, first-order expectations and second-order expectations are related to each other. Given the beliefs P_H, the Hider chooses A if $P_H < 0.5$, B if $P_H > 0.5$ and he is indifferent between A and B if $P_H = 0.5$. Therefore the Hider chooses the alternative to which her first-order beliefs attach the smallest probability. Similarly, let P_S denote the Seeker's first-order expectations, defined as the subjective probability that the Seeker assigns to the event that the Hider will choose alternative A. Thus, $1 - P_S$ are the Seeker's first-order beliefs that the Hider will choose alternative B. Given her beliefs P_S, the Seeker chooses A if $P_S > 0.5$, B if $P_S < 0.5$ and he is indifferent between A and B if $P_S = 0.5$. Therefore the Seeker chooses the alternative to which her first-order beliefs attach the largest probability.

Let us denote the Hider's best response to first-order beliefs $br_H(P_H)$ as the function:

$$br_H(P_H) = \begin{cases} A & \text{if } P_H < 0.5 \\ B & \text{if } P_H > 0.5 \\ A \text{ or } B & \text{if } P_H = 0.5 \end{cases} \tag{4}$$

where A or B denotes the situation in which the Hider is indifferent between A and B.[10] Similarly, let us denote the Seeker's best response to first-order beliefs $br_S(P_S)$ as the function:

$$br_S(P_S) = \begin{cases} A & \text{if } P_S > 0.5 \\ B & \text{if } P_S < 0.5 \\ A \text{ or } B & \text{if } P_S = 0.5 \end{cases} \tag{5}$$

where A or B denotes the situation in which the Seeker is indifferent between A and B.

Let us now define Hider's and Seeker's second-order expectations. Second-order expectations express a subject's expectations about the other subject's expectations. Let the Hider's and Seeker's second-order expectations be denoted by the subjective density functions q_H and q_S respectively. Let Q_H and Q_S denote the corresponding subjective cumulative distributions. Thus, $Q_H(x)$ denotes the subjective probability that the Hider assigns to the event that the Seeker's first-order beliefs P_S are smaller or equal than x. Similarly, $Q_S(x)$ denotes the subjective probability that the Seeker assigns to the event that the Hider's first-order beliefs P_H are smaller or equal than x.

What are the functions of best response to second-order expectations Q_H and Q_S? Lets illustrate the case of the Hider (the case of the Seeker is analogous). When the Hider holds first-order beliefs $P_H < 0.5$, she thinks that the probability of the Seeker choosing A is smaller than 0.5. The Hider's best-response function then prescribes choice A. Since the Seeker chooses A if her first-order beliefs is $P_S > 0.5$, the Hider's subjective probability of the Seeker choosing A corresponds to her subjective probability of the Seeker in turn holding first-order belief $P_S > 0.5$. Given the definition of second-order beliefs $Q_H(x)$, the Hider's subjective probability of the Seeker holding first-order belief $P_S > 0.5$

[10]Notice that saying that a subject is indifferent between A and B does not allow us to expect any specific likelihood of A or B actually been chosen. Attaching a likelihood of 0.5 to A and 0.5 to B would be arbitrary, since an indifferent subject can randomize between A and B with any probability weights.

is $1 - Q_H(0.5)$. If $1 - Q_H(0.5) < 0.5$, i.e. $Q_H(0.5) > 0.5$, the Hider's best response to her second-order beliefs is A. Therefore, we can rewrite the Hider's and Seeker's functions of best response to second-order beliefs, denoted $br_H(Q_H)$ and $br_S(Q_S)$ respectively, as:

$$br_H(Q_H) = \begin{cases} A & \text{if } Q_H(0.5) > 0.5 \\ B & \text{if } Q_H(0.5) < 0.5 \\ A \text{ or } B & \text{if } Q_H(0.5) = 0.5 \end{cases} \tag{6}$$

and

$$br_S(Q_S) = \begin{cases} A & \text{if } Q_S(0.5) > 0.5 \\ B & \text{if } Q_S(0.5) < 0.5 \\ A \text{ or } B & \text{if } Q_S(0.5) = 0.5. \end{cases} \tag{7}$$

When assessing how experimental choice data compares with the behavior prescribed by best response to first-order beliefs and best response to second-order beliefs, we employ the following concept of *consistency.* We define the Hider's observed choice C_H and first-order beliefs P_H as consistent if $C_H \in br_H(P_H)$ and the Seeker's observed choice C_S and first-order beliefs P_S as consistent if $C_S \in br_S(P_S)$. Analogously, we define the Hider's observed choice C_H and second-order beliefs Q_H as consistent if $C_H \in br_H(Q_H)$ and the Seeker's observed choice C_S and second-order beliefs Q_S as consistent if $C_S \in br_S(Q_S)$. Given the definition of the best-response functions in (4)-(7) and the definition of consistency between choice and (first-/second-order) beliefs, the elicitation of beliefs in probabilistic form is required to compare experimental choice data to the predictions of best-response behavior. The methods to elicit first- and second-order expectations, described in Section 2, enable us to measure the variables P_H, P_S, Q_H and Q_S.

How are the Hider's and the Seeker's first-order expectations P_H and P_S related to their second-order expectations Q_H and Q_S? When investigating the relationship between first- and second-order beliefs, we employ two different definitions of consistency, which we define as *belief-consistency* and *behavior-consistency.*

The concept of *belief-consistency* captures whether an agent thinks that her opponent best responds to his first-order beliefs. Specifically, *belief-consistency* holds for the Hider if she thinks that the Seeker makes his decision based on (5), i.e. choosing A if $P_S > 0.5$, choosing B if $P_S < 0.5$, and being indifferent between A and B if $P_S = 0.5$. Therefore, the Hider's first- and second-order beliefs are *belief-consistent* if the probability P_H, which the Hider assigns to the event that the Seeker chooses A, coincides with the probability $1 - Q_H(0.5)$, which the Hider assigns to the event that the Seeker considers A more likely to be chosen by the Hider (i.e., the event in which $P_S > 0.5$). Analogously, *belief-consistency* holds for the Seeker if she thinks that the Hider makes his decision based on (4), i.e. choosing A if $P_H < 0.5$, choosing B if $P_H > 0.5$, and being indifferent between A and B $P_H = 0.5$. Therefore, the Seeker's first- and second-order beliefs are *belief-consistent* if the probability P_S, which the Seeker assigns to the event that the Hider chooses A, coincides with the probability $Q_S(0.5)$, which the Seeker assigns to the event that the Hider considers A less likely to be chosen by the Seeker (i.e., the event in which $P_H < 0.5$). We can then give the following definition.

Definition A Hider's first- and second-order beliefs are *belief-consistent* if $P_H = 1 - Q_H(0.5)$. A Seeker's first- and second-order beliefs are *belief-consistent* if $P_S = Q_S(0.5)$.

The concept of *behavior-consistency* instead captures whether first- and second-order beliefs prescribe the same best-response behavior.

Definition A Hider's first- and second-order beliefs are *behavior-consistent* if $br_H(P_H) = br_H(Q_H)$. A Seeker's first- and second-order beliefs are *behavior-consistent* if $br_S(P_S) = br_S(Q_S)$.

While *belief-consistency* requires an exact equivalence $P_H = 1 - Q_H(0.5)$ for the Hider and $P_S = Q_S(0.5)$ for the Seeker, *behavior-consistency* only requires that first-order beliefs P and second-order beliefs $Q(0.5)$ either stand on the opposite (same) side of the 0.5 value for the Hider (Seeker) or stand both at the 0.5 value. Therefore, *behavior-consistency* requires that one of the following cases holds for the Hider: (i) $P_H > 0.5$ and $Q_H(0.5) < 0.5$, (ii) $P_H < 0.5$ and $Q_H(0.5) > 0.5$, or (iii) $P_H = 0.5$ and $Q_H(0.5) = 0.5$. For the Seeker, *behavior-consistency* requires that one of the following cases holds: (i) $P_S > 0.5$ and $Q_S(0.5) > 0.5$, (ii) $P_S < 0.5$ and $Q_S(0.5) < 0.5$, or (iii) $P_S = 0.5$ and $Q_S(0.5) = 0.5$. Moreover, *belief-consistency* implies *behavior-consistency*, while the opposite does not.

It is worth noting that two cases can never arise, whether we consider *belief-consistency* or *behavior-consistency* as a criterium. One case corresponds to the situation in which both consistency between choice and first-order beliefs and consistency between first- and second-order beliefs hold, while consistency between choice and second-order beliefs fails. Under *behavior-consistency*, this situation cannot occur (for example, for the Hider) because if $C_H \in Br(P_H)$ and $Br(P_H) = Br(Q_H)$, then it must be that $C_H \in Br(Q_H)$. Under *belief-consistency*, the described situation cannot occur because if $C_H \in Br(P_H)$ and $P_H = 1 - Q_H(0.5)$, then it must be that $C_H \in Br(Q_H)$. Analogous arguments hold for the Seeker. The other case, which cannot arise, is the situation in which both consistency between choice and second-order beliefs and consistency between first- and second-order beliefs hold, while consistency between choice and first-order beliefs fails. The arguments (under *behavior-consistency* and under *belief-consistency*) are analogous to the ones just presented above.[11]

Given the possibility of indifference in the best-response functions $Br(P)$ and $Br(Q)$, Section 4 assesses the consistency of observed choices and beliefs separately for the entire sample and for the subsample of observations for which first- and/or second-order beliefs rule out indifference.

We conclude this section arguing that, by defining second-order beliefs as subjective probability distributions over [0,100]% and by eliciting the probability mass that a subject assigns to several intervals in the range [0,100]% (as described in Section 2), we overcome the limitations that would otherwise arise in the analysis of consistency (both consistency of choice and second-order beliefs and consistency of first- and second-order beliefs), were second-order beliefs defined and elicited as point-forecasts instead of probabilistic forecasts. For example, given the definition of *behavior-consistency* between first- and second-order beliefs, if second-order beliefs were elicited as point-forecasts, it could occur that a player's probabilistic first-order beliefs and point-forecast second-order beliefs are not *behavior-consistent*, even though her probabilistic first-order beliefs and probabilistic second-order beliefs would be *behavior-consistent*, were second-order beliefs elicited as a probabilistic measure.[12]

[11] It is also worth noting what are feasible situations that may arise in experimental data. One situation is the one in which the observed choice is consistent both with first-order beliefs and second-order beliefs, while first- and second-order beliefs are not *behavior-consistent*. This situation arises, for example, when $C_H = A$, $P_H = 0.5$, $Q_H(0.5) = 0.9$, since $A \in Br(P_H)$, $A \in Br(Q_H)$ but $Br(P_H) \neq Br(Q_H)$ because $Br(P_H) = A$ or B while $Br(Q_H) = A$. Similarly, a feasible situation is the one in which the observed choice is consistent both with first-order beliefs and second-order beliefs, while first- and second-order beliefs are not *belief-consistent*. This situation arises, for example, when $C_H = A$, $P_H = 0.3$, $Q_H(0.5) = 0.9$, since $A \in Br(P_H)$, $A \in Br(Q_H)$ but $P_H \neq 1 - Q_H(0.5)$.

[12] Costa-Gomes and Weizsacker (2008) seem to point out this limitation when they acknowledge, referring to the

4 Results

In this section we present the main results. Section 4.1 is an overview of the participants and the treatments. In Section 4.2 we describe the choices that hiders and seekers make and the following outcomes. In Section 4.3 and 4.4, we describe the elicited first- and second-order expectations. In Section 4.5 we turn to verifying the consistency between choice, first- and second-order expectations.

4.1 The Participants and the Treatments

The experiment was conducted under 6 treatments, each one with a different order in which questions about choice, first- and second-order expectations were asked. Each session corresponds to a treatment. Each session was initially scheduled to have about 26 students signed up, expecting about 18-20 students to show up on time and participating. The sessions turned out to have 20, 16, 18, 16, 26, 18 participants respectively (as reported in table 1), for a total of 114 participants and 456 observations.

Table 1: Sessions and Treatments.

Session Name	**Number of Participants**	**Number of Rounds**	**Number of observations**	**Task Order**
C 1 2	20	4	80	choice, 1st-order beliefs, 2nd-order beliefs
C 2 1	16	4	64	choice, 2nd-order beliefs, 1st-order beliefs
1 C 2	18	4	72	1st-order beliefs, choice, 2nd-order beliefs
1 2 C	16	4	64	1st-order beliefs, 2nd-order beliefs, choice
2 C 1	26	4	104	2nd-order beliefs, choice, 1st-order beliefs
2 1 C	18	4	72	2nd-order beliefs, 1st-order beliefs, choice
all	114	4	456	

Table 16 (in Appendix B) reports the sample distribution of participants in each treatment/session according to their field of studies and gender.[13] Female make up for 60% of all participants. This ratio

unpublished version of their paper (which contained results on elicited second-order beliefs), that 'a limitation of the analysis of these second-order belief statements is that we elicited point estimates of players' second-order beliefs, and not unrestricted probabilistic second-order beliefs. This restriction, which was made for practical reasons, gives rise to the possibility that a fully consistent player's stated first-order belief is not a best response to her own stated second-order belief, complicating the discussion of consistency.' It is important to notice that Costa-Gomes and Weizsacker (2008), as well as Bhatt and Camerer (2005), use interchangeably the terms 'consistency' and 'best response' when referring to any of the relationships between (i) choice and first-order beliefs, (ii) choice and second-order beliefs and (iii) first- and second order beliefs. We instead use 'consistency' and 'best response' interchangeably only for the relationships between (i) choice and first-order beliefs and (ii) choice and second-order beliefs. We prefer not to use phrases such as 'first-order beliefs are a best response to second-order beliefs', given that we intend the concept of 'best response' uniquely as a possible attribute of choice. The concepts of belief-consistency and behavior-consistency are therefore not related to 'first-order beliefs being a best response to second-order beliefs'. Belief-consistency has no relation with the concept of best response, and behavior-consistency stands for the best response to first-order beliefs coinciding with the best response to second-order beliefs (and not to 'first-order beliefs being a best response to second-order beliefs'.)

[13] Fields of studies are categorized as social sciences, humanities, or sciences. Majors in the social sciences include: economics, social policy, human development and psychological services, learning and organizational change, political science, psychology, sociology, education. Majors in the humanities include: art history, art theory, theatre, art theory and practice, classics, english, spanish, philosophy, legal studies, anthropology, gender studies, history, communication

is stable across treatments, except for treatment 2C1, in which females are 38% of the participants. Participants with a major in the Social Sciences make up 44% of the participants, while participants with a major in the Sciences or in the Humanities make up 26% of the participants respectively. This ratio is stable across treatments, except for treatment 2C1, in which students majoring in the social sciences are 27% of the participants. Thus, treatment 2C1 happens to have a lower ratio of female participants and a lower ratio of social-sciences-major participants compared to other treatments.

4.2 Choices and Outcomes

Table 2 reports the sample frequency, in percentages, of subjects choosing alternative A, distinguishing between subjects with the role of hiders and subjects with the role of seekers. Over all periods and all treatments, there is a statistically significant difference (at the 0.1 percent level) in the frequency with which either hiders or seekers choose A: hiders choose A 60% of the time and seekers 41% of the time.[14] Over all treatments, there is not a trend of change across periods for hiders. For seekers, instead, we observe a statistically significant increase (at the 5 percent level) in the frequency with which they choose A, from 35% in period 1 to 54% in period 4.

Changing the order in which the tasks are presented seems to have no effect on hiders (the frequency with which they choose A is approximately 60% across all treatments), while it seems to have an effect on seekers. Specifically, when first-order beliefs are the first task to be presented, the frequency with which seekers choose A is higher than in the other cases (60% compared to 33%). This difference is also statistically significant at the 0.1 percent level.

Table 3 reports the sample frequency of choice outcomes, together with the sample frequency with which players with the role of Seeker win the prize, by successfully matching their opponent's choice. The most common outcome is (A, B), corresponding to the Hider choosing A and the Seeker choosing B, occurring in 36% of the Hider-Seeker matched observations. The least common outcome is (B, A), occurring in 17% of the Hider-Seeker matched observations. On average over all periods and all treatments, seekers win the prize in 48% of the cases. On average over all treatments, the percentage of cases in which seekers win the prize varies across time, from 40% in the first round to 49% in the last round, but the change is not statistically significant. On average over all periods, we do not find statistically significant differences across treatments. Results are omitted for brevity.[15]

4.3 First-Order Subjective Expectations

Table 17 (in Appendix B) reports the sample distribution of the answers given to the first-order beliefs task. Table 20 (also in Appendix B) shows that values of first-order expectations vary across observable characteristics of the subjects.

studies, journalism, comparative literary studies, middle eastern language and civilization, european studies, international studies, religious studies, music. Majors in the sciences include: biology, biochemistry, chemistry, environmental studies, mathematics, statistics, material science, chemical/civil/electrical/mechanical/computer/industrial engineering.

[14] In their hide-and-seek experiment with four identical boxes labeled, from left to right, A, B, A, A, Rubinstein, Tversky and Heller (1996) finds that the distribution of seekers' answers (13%, 31%, 45%, 11%) is strongly biased towards the central A box, avoiding the edges. Similarly, the distribution of hiders' answers (9%, 36%, 40%, 15%) is biased toward the central A box. The strong tendency to avoid the edges was also observed by Ayton and Falk (1995) in their experiment 'hide a treasure in a $5X5$ table', where the subject hides a treasure in one of the table's 25 boxes.

[15] Differences across treatments and periods cannot be evaluated statistically due to the small sample.

Table 2: Frequency of choice A.

	Period				
	1	**2**	**3**	**4**	**all**
	%	%	%	%	%
Hider					
12C	75	25	50	50	50
1C2	78	100	67	44	72
21C	33	56	67	67	56
2C1	54	46	62	62	56
C12	70	60	60	60	62
C21	50	75	62	75	66
all	60	60	61	60	60
Seeker					
12C	50	50	25	75	50
1C2	44	56	89	89	69
21C	33	22	0	22	19
2C1	31	46	46	46	42
C12	30	40	20	30	30
C21	25	12	38	75	38
all	35	39	37	54	41

Table 3: Frequency of choice outcomes.

	Period				
	1	**2**	**3**	**4**	**all**
	%	%	%	%	%
Choice outcome					
(A,A)	18	23	26	32	25
(A,B)	42	37	35	28	36
(B,A)	18	16	11	23	17
(B,B)	23	25	28	18	23
all	100	100	100	100	100
Seeker wins	40	47	54	49	48

Note: (X,Y)=(Hider's choice, Seeker's choice).

First-order expectations vary across field of studies. Participants with a Social Sciences major give more often the central answer of 50 (57.5% of the time) and less often extreme answers below 5 (6% of the time) or above 95 (7.5% of the time) compared to participants with a Humanities major (for which the corresponding percentages are 37.5%, 14.17% and 13.33%) or a Sciences major (for which the corresponding percentages are 38.33%, 10% and 13.33%).

First-order expectations vary also across gender. Female participants give more often the central answer of 50 (51.1% of the time) and less often extreme answers below 5 (7.35% of the time) or above 95 (7.72% of the time) compared to male participants (for which the corresponding percentages are 39.67%, 12.5% and 14.67%). Table 20 shows that most of these differences are statistically significant.

The effect of the field of studies persists when separating the sample according to gender. Results are omitted for brevity. Female participants with a major in the Social Sciences give more often the central answer of 50 (57.81% of the time) than female participants with a major in the Humanities, who choose 50 as an answer 44.32% of the time. This difference is statistical significant at the 5 percent level. Male participants with a major in the Social Sciences give more often the central answer of 50 (56.94% of the time) than male participants with a major in the Humanities or in the Sciences, who choose 50 as an answer 18.75% and 30.56% of the time respectively. These differences are significant at the 0.1 percent level.

A first-order belief equal to 50 percent implies that the subject is indifferent between A and B. As table 17 reports, first-order beliefs equal to 50 percent occur in almost half of the observations. What do subjects, who hold first-order beliefs that imply indifference between A and B, actually choose? Table 4 reveals that in the first period approximately 70% of hiders with beliefs $P_H = 0.5$ chooses A and 60% of seekers with beliefs $P_S = 0.5$ chooses B. In the fourth and last period, while players with the role of seekers appear equally likely to choose A or B, hiders still choose A more often.

Choice	Type Hider %	Seeker %	All %
first period (1)			
A	68	39	52
B	32	61	48
All	100	100	100
last period (4)			
A	64	52	59
B	36	48	41
All	100	100	100

Table 4: Choice made by subjects holding a first-order belief equal to 50 percent (i.e., assigning a 50 percent subjective probability to the event of the opponent choosing A).

Choice	Type Hider %	Seeker %	All %
first period (1)			
A	63	39	50
B	37	61	50
All	100	100	100
last period (4)			
A	53	48	51
B	47	52	49
All	100	100	100

Table 5: Choice made by subjects holding a second-order belief $Q(0.5) = 0.5$ (i.e., assigning a equal probability mass to the event of the opponent having a first-order belief lower or higher than 50 percent).

4.4 Second-Order Subjective Expectations

The experimental design elicits participants' second-order subjective beliefs (i.e., beliefs about opponent's beliefs) by asking two types of questions. First, participants are asked what they expect to be the most likely value of their opponent's answer to the first-order beliefs task. In other words, they are asked what they expect their opponent to report as the probability that they will choose alternative A. Second, participants are asked to place probabilities to the intervals [0,5]%, (5,20]%, (20,50]%, (50,80]%, (80,95]% and (95,100]% where each interval represents a possible range of values for the opponent's answer to the first-order beliefs task. In this section we describe first the answers to the question presented in the form of the *most likely value* and then the answers to the question in the form of *probabilities* assigned to intervals.

Table 18 (in Appendix B) reports the sample distribution of the answers given to the second-order beliefs task, which requires to report the *most likely value* of one's opponent's beliefs. The value of 50% is the most commonly reported answer, reported in approximately half of all observations. Table 21 (in Appendix B) shows that the reported *most likely value* varies across subjects' observable characteristics. First, answers vary across field of studies: participants whose major is in the Humanities give less often the central answer of 50 (46.67% of the time) and more often extreme answers below 5 (10.83% of the time) or above 95 (15.83% of the time) compared to participants whose major is in the Social Science (for which the corresponding percentages are 58.5%, 2% and 9.5%) . Second, answers vary also across gender: male participants give less often the central answer of 50 (46.2% of the time) and more often extreme answers above 95 (16.85% of the time) compared to female participants (for which the corresponding percentages are 54.17%, and 9.87%). Table 21 shows that most of these differences are statistically significant.

The effect of the field of studies persists when separating the sample according to gender. Results are omitted for brevity. Female participants with a major in the Humanities give more often extreme answers below 5 (11.36% of the time) and above 95 (10.23% of the time) than female participants with a major in the Social Sciences, who choose values below 5 as an answer 1.56% of the time and values above 95 3.12% of the time. These differences are statistically significant at the 1 and 5 percent level respectively. Male participants with a major in the Humanities give less often the central answer of 50 (25% of the time) than male participants with a major in the Social Sciences, who choose 50 as an answer 55.56% of the time. This difference is significant at the 1 percent level.

Table 6: Sample distribution of the number of intervals assigned with a positive probability and whether those intervals are adjacent to one another or not.

Number of intervals assigned with positive probability	**non adjacent intervals**	**adjacent intervals**	**all**
1			9%
2	4%	14%	18%
3	4%	7%	11%
4	3%	23%	26%
5	1%	6%	7%
6			28%

We now turn to describe the answers to the elicitation task that was completed by assigning *probabilities* to intervals. Table 6 reports the sample distribution of the number of intervals assigned with positive probability. When applicable (i.e., for a number of intervals larger than the minimum 1 and smaller than the maximum 6), the table also distinguishes whether the intervals assigned with positive probability are adjacent to one another or not. The most common answers consist of assigning positive probability to six, four or two intervals (28%, 26% and 18% of observations). Moreover, subjects more often assign positive probability to adjacent intervals than to non adjacent intervals. Of the 285 observations in which positive probability is assigned to 2-5 intervals, 232 (81%) correspond to adjacent intervals.

Table 19 (in Appendix B) reports the sample distribution of the value of x such that the subjective cumulative distribution function, which expresses second-order beliefs, evaluated at 0.50 equals x, i.e. $Q(0.5) = x$. Being 50% the right endpoint of one of the intervals presented to the experimental subjects, the value of x is readily available from the elicited data. As illustrated in Section 3, a Hider best responding to her second-order beliefs is indifferent between A and B when $Q_H(0.5) = 0.5$ and analogously a Seeker best responding to her second-order beliefs is indifferent between A and B when $Q_S(0.5) = 0.5$. As table 19 reports, second-order beliefs with $Q(0.5) = 0.5$ occur in approximately 40 percent of the observations. What do subjects, who hold second-order beliefs that imply indifference between A and B, actually choose? Table 5 reveals that in the first period approximately 60% of hiders holding beliefs $Q_H(0.5) = 0.5$ chooses A and 60% of seekers holding beliefs $Q_S(0.5) = 0.5$ chooses B. In the fourth and last period, both hiders and seekers holding beliefs $Q_S(0.5) = 0.5$ choose A or B equally often.

In Appendix A we perform both a nonparametric and a parametric analysis to assess the coherence of the answers in terms of *most likely value* and the answers in terms of *probabilities* assigned to intervals.

4.5 Consistency of Choice, First-Order and Second-Order Expectations

In this section we investigate the degree to which choice, first- and second-order beliefs are consistent with each other. Table 7 presents a summary of the results.

Whenever the assessment of consistency between first- and second-order beliefs is involved, we perform it according to both criteria introduced in Section 3: *belief-consistency* and *behavior-consistency*.[16] Moreover, the assessment of *belief-consistency* is performed both using the definition presented in Section 3, which requires an exact equivalence $P_H = 1 - Q_H(0.5)$ or $P_S = Q_S(0.5)$, and two less stringent definitions, which don't require an exact equivalence and that may therefore alleviate the effect of participants rounding their subjective probabilities up or down. 5%-*belief-consistency* is satisfied if $|P_H - (1 - Q_H(0.5))| \leq 0.05$ or $|P_S - Q_S(0.5)| \leq 0.05$, and 10%-*belief-consistency* is satisfied if $|P_H - (1 - Q_H(0.5))| \leq 0.10$ or $|P_S - Q_S(0.5)| \leq 0.10$. We relabel the three conditions as 0%-*belief-consistency*, 5%-*belief-consistency* and 10%-*belief-consistency* respectively.

Choice and first-order beliefs are consistent with each other in 89.5% of the observations.[17] Choice and second-order beliefs are consistent with each other 75.4% of the observations.[18] Results on the consistency between first- and second-order beliefs differ whether we consider *belief-consistency* or *behavior-consistency* as a criterium: 0%-*belief-consistency* holds in 33.6% of observations while *behavior-consistency* holds in 47.4% of observations. Assessing whether all three conditions hold simultaneously (consistency of choice and first-order beliefs, consistency of choice and second-order beliefs, and consistency of first- and second-order beliefs), the criterium of 0%-*belief-consistency* leads to a percentage of 32.7% of observations, while the criterium of *behavior-consistency* leads to a percentage of 44.1% of observations.

The analysis presented in Section 3 has shown the effect that the specific values $P = 0.5$ (for first-order beliefs) and $Q(0.5) = 0.5$ (for second-order beliefs) have on decision-making. First-order beliefs $P = 0.5$ imply that a subject best responding to her first-order beliefs is indifferent between A and B. Indifference then implies that any observed choice is consistent with first-order beliefs. Analogously, second-order beliefs $Q(0.5) = 0.5$ imply that a subject best responding to her second-order beliefs is indifferent between A and B. Indifference then implies that any observed choice is consistent with second-order beliefs. Moreover, Sections 4.3 and 4.4 have revealed that for a large fraction of elicited beliefs $P = 0.5$ and $Q(0.5) = 0.5$.

Indifference, in the form of $P = 0.5$ and/or $Q(0.5) = 0.5$, affects not only the assessment of consistency between choice and first-order beliefs and between choice and second-order beliefs, but also the assessment of consistency between first- and second-order beliefs, under both criteria of

[16] As argued in Section 3, two cases in the assessment of consistency between choice, first- and second-order beliefs can never arise (either under *belief-consistency* or *behavior-consistency*). One case corresponds to the situation in which both consistency between choice and first-order beliefs and consistency between first- and second-order beliefs hold, while consistency between choice and second-order beliefs fails. The other case corresponds to the situation in which both consistency between choice and second-order beliefs and consistency between first- and second-order beliefs hold, while consistency between choice and first-order beliefs fails.

[17] 86.8% of the observations of hiders and 92.1% of the observations of seekers.

[18] 79.4% of the observations of hiders and 71.5% of the observations of seekers.

Table 7: Consistency of choice, first- and second-order expectations. Sample frequency in %. The first panel reports sample frequencies computed over the entire sample. The remaining panels divide the data in subsamples according to whether first-order and/or second-order beliefs imply or not indifference between choosing A or B. For each panel, sample frequencies are computed for each treatment.

	choice and 1st-order beliefs	**choice and 2nd-order beliefs**	**1st- and 2nd-order beliefs**				**choice, 1st- and 2nd-order beliefs**				**obs**
			belief-consistency 0%	5%	10%	*behavior-consistency*	*belief-consistency* 0%	5%	10%	*behavior-consistency*	
all obs											
12C	85.9	79.7	48.4	59.4	62.5	62.5	45.3	53.1	54.7	54.7	64
1C2	97.2	62.5	22.2	26.4	40.3	33.3	22.2	26.4	38.9	31.9	72
21C	86.1	79.2	26.4	30.6	44.4	50.0	26.4	30.6	40.3	45.8	72
2C1	88.5	72.1	31.7	40.4	52.9	45.2	30.8	38.5	45.2	42.3	104
C12	85.0	86.2	41.2	45.0	60.0	47.5	41.2	43.8	52.5	45.0	80
C21	95.3	73.4	32.8	42.2	50.0	48.4	31.2	39.1	43.8	46.9	64
all treatments	89.5	75.4	33.6	40.4	51.8	47.4	32.7	38.4	45.8	44.1	456
only obs with $P \neq 0.5$, $Q(0.5) \neq 0.5$											
12C	70.8	58.3	33.3	45.8	45.8	70.8	25.0	33.3	33.3	50.0	24
1C2	91.7	37.5	4.2	16.7	25.0	37.5	4.2	16.7	25.0	33.3	24
21C	73.7	68.4	13.2	15.8	34.2	57.9	13.2	15.8	28.9	50.0	38
2C1	85.7	50.0	16.7	21.4	31.0	50.0	14.3	19.0	26.2	42.9	42
C12	68.8	68.8	31.2	50.0	50.0	62.5	31.2	43.8	43.8	50.0	16
C21	89.7	62.1	24.1	37.9	44.8	58.6	20.7	34.5	41.4	55.2	29
all treatments	80.9	57.2	19.1	28.3	37.0	55.5	16.8	24.9	31.8	46.8	173
only obs with $P \neq 0.5$, $Q(0.5) = 0.5$											
12C	71.4	100	0	0	14.3	0	0	0	14.3	0	7
1C2	100.0	100	0	0	33.3	0	0	0	33.3	0	15
21C	100.0	100	0	40.0	60.0	0	0	40.0	60.0	0	5
2C1	62.5	100	0	31.2	62.5	0	0	31.2	43.8	0	16
C12	65.0	100	0	0	45.0	0	0	0	35.0	0	20
C21	100	100	0	0	0	0	0	0	0	0	8
all treatments	78.9	100	0	9.9	39.4	0	0	9.9	32.4	0	71
only obs with $P = 0.5$, $Q(0.5) \neq 0.5$											
12C	100	70.0	0	40.0	50.0	0	0	30	30.0	0	10
1C2	100	33.3	0	0	16.7	0	0	0	11.1	0	18
21C	100	80.0	0	0	13.3	0	0	0	6.7	0	15
2C1	100	60.0	0	10.0	30.0	0	0	5.0	15.0	0	20
C12	100	62.5	0	0	18.8	0	0	0	0	0	16
C21	100	53.8	0	15.4	38.5	0	0	7.7	15.4	0	13
all treatments	100	58.7	0	8.7	26.1	0	0	5.4	12.0	0	92
only obs with $P = 0.5$, $Q(0.5) = 0.5$											
all treatments	100	100	100	100	100	100	100	100	100	100	120

behavior-consistency and *belief-consistency*. First-order beliefs $P = 0.5$ matched with second-order beliefs $Q(0.5) = 0.5$ trivially satisfy both *behavior-consistency* and *belief-consistency*. Instead, first-order beliefs $P = 0.5$ matched with second-order beliefs $Q(0.5) \neq 0.5$ rule out *behavior-consistency* and *belief-consistency*. Analogously, second-order beliefs $Q(0.5) = 0.5$ matched with first-order beliefs $P \neq 0.5$ also rule out *behavior-consistency* and *belief-consistency*.

The inclusion of observations with $P = 0.5$ and/or $Q(0.5) = 0.5$ in the upper panel of table 7 affects the results in two ways. First, it affects the assessment of consistency between choice and beliefs (first- or second-order beliefs), i.e. whether observed choice corresponds to best-response to elicited beliefs. Second, it affects the assessment of consistency between first- and second-order beliefs (independently of whether observed choice corresponds to best-response to elicited beliefs). The lower panels of table 7 illustrate how results change.

Lets start from the third panel. Observation in the third panel of table 7 (corresponding to first-order beliefs $P \neq 0.5$ matched with second-order beliefs $Q(0.5) = 0.5$) trivially satisfy consistency between choice and second-order beliefs but at the same time rule out *behavior-consistency* and 0%-*belief-consistency*. Observation in the fourth panel of table 7 (corresponding to first-order beliefs $P = 0.5$ matched with second-order beliefs $Q(0.5) \neq 0.5$) trivially satisfy consistency between choice and first-order beliefs but at the same time rule out *behavior-consistency* and 0%-*belief-consistency*. Observations in the last panel of table 7 (corresponding to first-order beliefs $P = 0.5$ matched with second-order beliefs $Q(0.5) = 0.5$) trivially satisfy all conditions.

Lets now turn to the second panel of table 7, which corresponds to the subsample for which indifference both in terms of first-order beliefs and in terms of second-order beliefs is ruled out ($P \neq 0.5$ and $Q(0.5) \neq 0.5$). This represents the most relevant subsample in order to argue about consistency without the results being influenced by indifference in best-response. Compared with the entire sample (see first panel), consistency between choice and first-order beliefs falls from 89.5% to 80.9%, and consistency between choice and second-order beliefs falls from 75.4% to 57.2%. Sample frequencies decrease also in terms of *belief-consistency*: under 0%-*belief-consistency* consistency between first- and second-order beliefs decreases from 33.6% to 19.1% and simultaneous consistency between (i) choice and first-order beliefs, (ii) choice and second-order beliefs, (iii) first- and second-order beliefs decreases from 32.7% to 16.8%. However, sample frequencies do *increase* (slightly) in terms of *behavior-consistency*: consistency between first- and second-order beliefs increases from 47.4% to 55.5% and simultaneous consistency between (i) choice and first-order beliefs, (ii) choice and second-order beliefs, (iii) first- and second-order beliefs increases from 44.1% to 46.8%.

Tables 8 and 9 report more detailed results on the subsample of observations with both $P \neq 0.5$ and $Q(0.5) \neq 0.5$. Since the definitions of (i) consistency between choice and first-order beliefs and of (ii) consistency between choice and second-order beliefs are the same in both tables, the reported totals over rows and over columns are the same in both tables. Both tables report results across all observations in the upper panel, results across all hiders-observations in the middle panel, and results across all seekers-observations in the bottom panel. Looking at table 9 we can notice that, independently from the role played in the game, the two most frequent cases are: (i) first- and second-order beliefs are *behavior-consistent* and choice is consistent with both, (ii) first- and second-order beliefs are not *behavior-consistent* and choice is consistent with first-order beliefs only. Less frequent cases are: (iii) first- and second-order beliefs are *behavior-consistent*, but choice is not consistent with either, (iv) first- and second-order beliefs are not *behavior-consistent* and choice is consistent with second-order beliefs only.

Table 8: Consistency of choice, first- and second-order beliefs. Sample frequency in %. Consistency between first- and second-order beliefs is defined as *belief-consistency*. Sample restricted to observations with both $P \neq 0.5$ and $Q(0.5) \neq 0.5$.

all	**choice and 2nd-order beliefs**		
	not consistent	consistent	
choice and 1st-order beliefs			
not consistent			19.1
1st- and 2nd-order beliefs			
not consistent	6.4	10.4	
consistent	2.3		
consistent			80.9
1st- and 2nd-order beliefs			
not consistent	34.1	30.1	
consistent		16.8	
	42.8	57.2	
hider	**choice and 2nd-order beliefs**		
	not consistent	consistent	
choice and 1st-order beliefs			
not consistent			
1st- and 2nd-order beliefs			24.7
not consistent	10.6	10.6	
consistent	3.5		
consistent			75.3
1st- and 2nd-order beliefs			
not consistent	23.5	36.5	
consistent		15.3	
	37.6	62.4	
seeker	**choice and 2nd-order beliefs**		
	not consistent	consistent	
choice and 1st-order beliefs			
not consistent			13.6
1st- and 2nd-order beliefs			
not consistent	2.3	10.2	
consistent	1.1		
consistent			
1st- and 2nd-order beliefs			86.4
not consistent	44.3	23.9	
consistent		18.2	
	47.7	52.3	

Table 9: Consistency of choice, first- and second-order beliefs. Sample frequency in %. Consistency between first- and second-order beliefs is defined as *behavior-consistency*. Sample restricted to observations with both $P \neq 0.5$ and $Q(0.5) \neq 0.5$.

all	**choice and 2nd-order beliefs**		
	not consistent	consistent	
choice and 1st-order beliefs			
not consistent			19.1
1st- and 2nd-order beliefs			
not consistent		10.4	
consistent	8.7		
consistent			80.9
1st- and 2nd-order beliefs			
not consistent	34.1		
consistent		46.8	
	42.8	57.2	
hider	**choice and 2nd-order beliefs**		
	not consistent	consistent	
choice and 1st-order beliefs			
not consistent			
1st- and 2nd-order beliefs			24.7
not consistent		10.6	
consistent	14.1		
consistent			75.3
1st- and 2nd-order beliefs			
not consistent	23.5		
consistent		51.8	
	37.6	62.4	
seeker	**choice and 2nd-order beliefs**		
	not consistent	consistent	
choice and 1st-order beliefs			
not consistent			13.6
1st- and 2nd-order beliefs			
not consistent		10.2	
consistent	3.4		
consistent			86.4
1st- and 2nd-order beliefs			
not consistent	44.3		
consistent		42.0	
	47.7	52.3	

Note: Ruling out indifference implies that, if 1st- and 2nd-order beliefs are not *behavior-consistent*, one belief must prescribe best response A and the other best response B. Thus, observed choice is necessarily consistent with either 1st- or 2nd-order beliefs. Therefore, two additional cells in the table (in additional to the two already mentioned in footnote 15) are necessarily empty. Ruling out indifference does not have this effect if the criterium of *belief-consistency* is employed (see Table 8).

It is interesting to compare the results presented above with the results presented by Bhatt and Camerer (2005). They report results from a series of experiments using several matrix games (solvable by 2 or 3 steps of iterated deletion of dominated strategies), in which participants' first- and second-order beliefs are recorded together with their choices. Both first- and second-order beliefs are measured non-probabilistically. In a game with action space {A,B}, subjects are required to report the action that they expect their opponent to choose (for first-order beliefs) or that they expect their opponent to think that *they* will choose (for second-order beliefs).[19]

Choice and beliefs data are then used to check whether equilibrium conditions hold. Eliciting first- and second-order beliefs in terms of actions, equilibrium conditions are defined as (i) choice being a best response to first-order beliefs, (ii) first-order beliefs being a best response to second-order beliefs, and (iii) choice being a best response to second-order beliefs. Over all the games, condition (i) holds in 66% of the trials, condition (ii) in 63% of the trials, condition (iii) in 75% of the trials and all conditions hold simultaneously in 23% of the trials. Therefore, the equilibrium requirement between choice and beliefs holds more often for second-order beliefs than for first-order beliefs (75% versus 66%).[20,21]

We are aware that the comparison between our results and those of Bhatt and Camerer (2005) is not unambiguous, both given to the fact that the games employed are different, and given to the fact that beliefs in our dataset are probabilistic while those in Bhatt and Camerer (2005) are not. By defining both first- and second-order beliefs probabilistically, we introduced two concepts of consistency (*belief-consistency* and *behavior-consistency*) which cannot be applied to the dataset of Bhatt and Camerer (2005). Therefore, we don't compare the results in terms of the relationship between first- and second-order beliefs. However, we can compare the results in terms of the relationship between observed choice and first- or second-order beliefs, since the criterium employed in both papers is the same: check whether observed choice coincides with the best response to beliefs. In our experiment, in contrast with Bhatt and Camerer (2005), the frequency with which consistency is achieved between choice and second-order beliefs is lower than the frequency with which consistency is achieved between choice and first-order beliefs.

We conjecture that the difference between the two sets of the results is likely to be related to the

[19] The format is of the type: 'I think that the opponent will do A' (for first-order beliefs) and 'I think that the opponent believes that I will do B' (for second-order beliefs). Compensation is given for right guesses.

[20] Bhatt and Camerer (2005) find these results surprising since they contrast the conjecture, which they describe in the following words: 'a natural intuition is that as players reason further up the hierarchy from choices, to beliefs, to iterated beliefs, their beliefs become less certain. Therefore, 2nd-order beliefs should be less consistent with [1st-order] beliefs than [1st-order] beliefs are with choices, and 2nd-order beliefs and choices should be least consistent'. Notice that the conjecture implicitly includes two sub-conjectures: (a) second-order beliefs are more uncertain than first-order beliefs and (b) under more uncertain beliefs it is less likely that equilibrium conditions will hold. However, neither (a) nor (b) can be tested using the experimental data of Bhatt and Camerer (2005) since in the experimental design beliefs are measured non-probabilistically.

[21] Finally, evidence that the frequency with which choice is consistent with 2nd-order beliefs is actually higher than the frequency with which 1st-order beliefs are consistent with 2nd-order beliefs (75% versus 63%) is interpreted by Bhatt and Camerer (2005) as 'a hint that the process of generating a [...] iterated belief might be similar to the process of generating a choice, rather than simply iterating a process of forming beliefs to guess what another player believes about oneself'. They argue, supported by evidence from the fMRI, that forming second-order beliefs is a mixture of forming beliefs and making choices. They find that forming second-order beliefs, compared to forming first-order beliefs, activates the *anterior insula* region of the brain, which previous studies have shown to be activated by a sense of agency and self-causation. Thus, the activity recorded by the fMRI when second-order beliefs are reported appears to be consistent with people anchoring on their own likely choice and then guessing whether other players will figure their choice out.

difference between the ways in which beliefs are measured: non-probabilistically in Bhatt and Camerer (2005) and probabilistically in our paper. Moreover, the effect that indifference may have on observed choices (both for the best response to first-order beliefs and for the best response to second-order beliefs) goes undetected in Bhatt and Camerer (2005), where experimental subjects cannot express any uncertainty when reporting their beliefs.

Table 10: Uncertainty and consistency between choice and expectations. Uncertainty is measured by the number of intervals over which a positive probability is placed.

	choice and 1st-order beliefs	**choice and 2nd-order beliefs**	**1st- and 2nd-order beliefs**				**choice, 1st- and 2nd-order beliefs**				**obs**
			belief-consistency			*behavior-consistency*	*belief-consistency*			*behavior-consistency*	
			0%	5%	10%		0%	5%	10%		
all obs											
intervals											
1	93.0	48.8	30.2	30.2	30.2	37.2	25.6	25.6	25.6	32.6	43
2	90.5	85.7	51.2	53.6	59.5	52.4	51.2	53.6	54.8	51.2	84
3	88.0	74.0	20.0	22.0	32.0	30.0	18.0	20.0	28.0	28.0	50
4	88.3	80.0	38.3	49.2	64.2	54.2	37.5	45.8	58.3	48.3	120
5	96.8	51.6	0.0	9.7	29.0	35.5	0.0	6.5	12.9	35.5	31
6	87.5	79.7	32.0	41.4	55.5	50.8	32.0	40.6	50.0	47.7	128
all	89.5	75.4	33.6	40.4	51.8	47.4	32.7	38.4	45.8	44.1	456
only obs with $P \neq 0.5$, $Q(0.5) \neq 0.5$											
intervals											
1	88.0	60.0	52.0	52.0	52.0	64.0	44.0	44.0	44.0	56.0	25
2	77.8	50.0	33.3	33.3	44.4	38.9	33.3	33.3	33.3	33.3	18
3	78.3	56.5	21.7	26.1	39.1	43.5	17.4	21.7	34.8	39.1	23
4	72.1	60.5	20.9	32.6	48.8	65.1	18.6	25.6	39.5	48.8	43
5	95.0	60.0	0.0	10.0	15.0	55.0	0.0	10.0	15.0	55.0	20
6	81.8	54.5	0.0	18.2	22.7	54.5	0.0	18.2	22.7	45.5	44
all	80.9	57.2	19.1	28.3	37.0	55.5	16.8	24.9	31.8	46.8	173

In the last part of this section we address the conjecture, present in Bhatt and Camerer (2005), that under more uncertain beliefs it is less likely that consistency between choice and beliefs will hold. The conjecture could be interpreted in two different ways: (a) second-order beliefs are more uncertain than first-order beliefs and for this reason consistency between choice and second-order beliefs is less likely to hold than consistency between choice and first-beliefs. (b) the more uncertain second-order beliefs are, the less likely it is that consistency between choice and second-order beliefs holds. Given that in this paper first-order beliefs are defined as a subjective probability value and second-order beliefs as a subjective probability distribution, it is not unambiguous how a comparison between the uncertainty of first-order beliefs and the uncertainty of second-order beliefs should be performed. Specifically, if second-order beliefs are a probability distribution and therefore its uncertainty can be measured by the dispersion of the probability distribution, then how can the uncertainty of first-order beliefs, which are simply a probability value, be defined? Conversely, if we allowed a subject to express her first-order beliefs not as a single probability value but as a probability distribution, thus recovering a measure which could represent the uncertainty of first-order beliefs, then what would be the appropriate way to elicit second-order beliefs? It would not seem appropriate anymore to elicit second-order beliefs as a single probability distribution. Allowing for multiple probability distributions, and therefore for ambiguity, would seem more reasonable. Therefore, we restrict our attention to uncertainty of second-order beliefs and verify whether under more uncertain second-order beliefs it is less likely that consistency between choice and second-order beliefs will hold.

As a measure for the uncertainty that a subject perceives, we use the number of intervals over which she places a positive probability, with the interpretation that if a subject places positive probability on a larger number of intervals, she expresses second-order beliefs that are more uncertain than those of a subject who places positive probability on a smaller number of intervals.[22] As table 10 shows, we don't find evidence that observations characterized by a larger number of intervals assigned with positive probability are characterized by a lower sample frequency of consistency between choice and second-order beliefs.[23,24]

5 Conclusion

This paper proposes a method to elicit probabilistic second-order expectations, along with first-order expectations, and examines the results collected in a Hide-and-Seek experiment in which players with the role of hiders and seekers make choices and answer both a first-order beliefs question and a second-order beliefs question. The proposed method consists of two steps. In the first step, subjects report what they think the most likely value is for their opponent's answer to the first-order beliefs question. In the second step, subjects report the probabilities with which their opponent's answer will fall within several intervals. We show that the coherence of the forecast in terms of the most likely value and the forecast in terms of probabilities over intervals is supported by both nonparametric and parametric analysis.

By measuring second-order beliefs probabilistically, not only do we find results in contrast with previous experimental work that uses non-probabilistic beliefs, but we can also assess hypotheses that were before unverifiable. First, in stark contrast with previous non-probabilistic evidence, we find that (i) consistency between choice and second-order beliefs occurs less often, and not more often,

[22] An alternative method could consist of measuring the uncertainty that a subject perceives by the interquartile range of the fitted distribution of her second-order beliefs. While we explored the use of this criterium, we prefer the use of the number of intervals assigned with positive probability, because it does not require to manipulate the data in any way. See Appendix A for details on the parametric fitting of second-order beliefs. Results using the interquartile range of the fitted distribution do not differ. Results are omitted for brevity.

[23] Nor we find any difference in the sample frequency of consistency between choice and first-order beliefs.

[24] We do find difference in assessing the consistency between first- and second-order beliefs. Under *belief-consistency*, we find that for those observations for which positive probabilities were assigned over a smaller number of intervals also corresponds to the observations for which the frequency of *belief-consistency* is higher. The same observation does not hold under *behavior-consistency*.

than consistency between choice and first-order beliefs and (ii) consistency between first- and second-order beliefs is difficult to achieve. Second, probabilistic second-order beliefs provide a measure of the uncertainty that subjects perceive in their expectations. Thus, we investigate the relationship between the uncertainty in second-order beliefs and the consistency between choice and second-order beliefs. Contrary to what has been conjectured in previous work, we do not find evidence that more uncertain expectations make it less likely for choice and expectations to be consistent with each other.

The results of this paper provide encouraging evidence in favor of the feasibility of measuring second-order beliefs probabilistically. Measuring second-order beliefs probabilistically represents a step forward in understanding the process of thinking that subjects experience when facing a strategic situation, and in turning the game-theoretic concept of higher-order beliefs into an observable variable.

While this paper documents the ability of subjects to report their subjective expectations in probabilistic form, we are aware that an alternative to the study of decision under uncertainty, and to the corresponding focus on subjective expectations, is represented by the study of decision under ambiguity. In decision under ambiguity subjects do not hold a unique subjective distribution for an unknown event but may hold a set of subjective distributions. We consider exploring the elicitation of beliefs under ambiguity an interesting topic for further research.

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Appendix A
Comparison between point and probabilistic second-order beliefs

In this appendix we describe a comparison between the two measures for second-order beliefs elicited in the experiment: the point prediction expressed by means of the 'most likely value' and the subjective probability distribution expressed by means of probabilities over intervals. We refer to Engelberg, Manski and Williams (2009) for an introduction of how point predictions and subjective probability distributions compare. While Engelberg, Manski and Williams (2009) studied the first-order subjective expectations of professional forecasters, the parametric and nonparametric analysis, which they proposed, can be readily extended to the present analysis of second-order subjective expectations of laboratory experimental subjects.

As argued in the paper, we can interpret the coherence of point prediction and subjective probability distribution as the elicited 'most likely value' being a measure of central tendency for the subjective probability distribution. Which measure of central tendency though? The mode? The mean? The median? While the actual wording used in the experiment (the 'most likely value') may hint that that the mode of the subjective probability distribution is in fact the measure elicited from the experimental subjects, we argue that we cannot make any inference on the subjective mode.

The limitation stems from the way the data for the subjective probability distribution was elicited. The experimental setting constrained to elicit probabilities only within certain intervals. Being the mode a local concept, we cannot tell which interval it is in. On the one hand, assuming that the mode is in the interval assigned with the highest probability is not a reasonable assumption, because the intervals have different widths. On the other hand, assuming that the density is uniform within each interval is not a reasonable assumption either, because the middle intervals have a large width. Therefore, we cannot make any inference on the subjective mode, and focus instead on inference on the subjective mean and median.

Nonparametric Analysis

In this section we use the elicited probabilities assigned to each interval [0,5]%, (5,20]%, (20,50]%, (50,80]%, (80,95]% and (95,100]% to compute bounds on the mean and median of the subjective distribution representing a subject's second-order beliefs.

Suppose that a subject assigns probability 0.30 to interval (20,50]%, 0.60 to interval (50,80]%, 0.05 to interval (80,95]%, 0.05 to interval (95,100]% and zero probability to all other intervals. Then we can conclude that the subjective median lies in the interval (50,80]%. Lower and upper bounds on subjective means are computed by placing the probability mass assigned to each interval at the interval's lower and upper endpoints respectively. In our example the lower bound is $0.30 \times 20\% + 0.60 \times 50\% + 0.05 \times 80\% + 0.05 \times 95\% = 44.75\%$ and the upper bound is $0.30 \times 50\% + 0.60 \times 80\% + 0.05 \times 95\% + 0.05 \times 100\% = 72.75\%$ The resulting bounds are (44.75, 72.75]%.

Table 11 reports the 25th, 50th and 75th percentiles of the sample distribution of the width of the bounds. The width of the bounds is closely dependent on the specific definition of the intervals used in the experiment. Specifically, the width of the bounds on the subjective median is usually 30 because the lower and upper bounds are usually 20 or 50 and 50 or 80, respectively. No stark difference stands out either across periods or across treatments. Results are omitted for brevity.

Table 11: Sample distribution of the width of the bounds on the subjective mean and median. Percentage points.

width	25th perc.	50th perc.	75th perc.
bounds on subj. mean	21	25	28
bounds on subj. median	30	30	30

Table 12 reports the percentage of observations for which the elicited 'most likely value' lies within the bounds on the subjective mean (69%) or within the bounds on the subjective median (79%). For most observations, the elicited 'most likely value' is consistent with the hypothesis that subjects report their subjective mean or median belief. The fact that the elicited 'most likely value' lies more often within the bounds on the subjective median than within the bounds on the subjective mean (79% versus 69%) is possibly due to the bounds on the subjective median being wider than the bounds on the subjective mean. Therefore, we cannot interpret this result as evidence in favor to the argument that subjects report their subjective median versus their subjective mean.

Table 12: Percentage of observations for which the elicited *most likely value* lies within the bounds on the subjective mean or within the bounds on the subjective median.

	within the bounds on the subj. mean %	within the bounds on the subj. median %	obs.
all obs.	69	79	456
by treatment			
12C	75	81	64
1C2	75	86	72
21C	60	67	72
2C1	66	76	104
C12	79	81	80
C21	61	83	64
by treatment			
21C and 2C1	64^{*}	72^{**}	176
other	73	83	280

Note: Benchmark category in Treatment is 'other'. *** denotes significantly different from the benchmark category at 0.1 percent, ** denotes significantly different from the benchmark category at 1 percent, * denotes significantly different from the benchmark category at 5 percent.

Third, the treatments in which subjects are asked to report their second-order beliefs as their first task, labeled 21C and 2C1, correspond to the treatments in which coherence with respect to the bounds on the subjective mean and median occurs less compared to all other treatments. (64% vs

73% for the subjective mean and 72% vs. 83% for the subjective median). The differences between the two classes of treatments are statistically significant at the 5 percent level (for the subjective mean) or at the 1 percent level (for the subjective median), as table 12 reports.

This result could be due to a difference in the width of bounds in treatments 21C and 2C1 compared to all other treatments. A less spread-out subjective distribution, characterized by narrower bounds on subjective mean and median, could make it less likely for the 'most likely value' to fall inside the bounds. This explanation however can be ruled out, since we find that the width of the bounds does not vary significantly across treatments. Thus, it could be the case that treatments other than 21C and 2C1 lead to a more thoughtful response of the second-order beliefs task because this is not the first task, and subjects have already gone through a choice task or a forecast task (or both). In turn, a more thoughtful response of the second-order beliefs task could make it more likely for the elicited 'most likely value' to fall inside the bounds.

Finally, the percentage of observations for which the elicited 'most likely value' lies within the bounds on the subjective mean or median does not vary significantly across periods nor across player roles (hider and seeker). Results are omitted for brevity.

Parametric Analysis

In this section we use the elicited probabilities assigned to each interval [0,5], (5,20], (20,50], (50,80], (80,95] and (95,100] to fit the subjective cumulative distribution functions (CDF) Qs, which represent subjects' second-order beliefs, i.e., subjects' beliefs about their opponents' beliefs. From the knowledge of the probabilities assigned to each interval, the value of the subjective CDF at the right endpoints of the six intervals can be inferred. In what follows, instead of using percentage points between 0 and 100, lets use values between 0 and 1. Therefore, lets relabeled the right endpoints of the six intervals as $r_1 = 0.05$, $r_2 = 0.20$, $r_3 = 0.50$, $r_4 = 0.80$, $r_5 = 0.95$ and $r_6 = 1$.

Lets denote the values of the subjective CDF at these points as $Q(r_1), ..., Q(r_6)$. Finally, lets denote the lower bound of the first interval and the upper bound of the last interval over which a subject places positive probability as L and R, respectively. Therefore, we denote the support of the subjective distribution as $[L, R]$.

Following Engelberg, Manski and Williams (2009), we maintain the assumption the subjective distribution is a member of the generalized Beta family, provided that it is possible to fit a unique Beta distribution to the data. It is possible to fit a unique Beta distribution to the data only when a subject assigns positive probability to at least three intervals. As reported in table 13, this occurs in 329 out of 456 observations (72% of observations)[25].

In the remaining 127 observations, positive probability is assigned to one or two intervals. In the cases with one interval (43 observations, 9% of the total), we assume that the subjective distribution has the shape of an isosceles triangle whose base coincides with the interval. In the cases with two intervals (84 observations, 18% of the total), the two are adjacent to one another in 64 observations and non-adjacent in 20 observations.

Consider the case of two adjacent intervals being assigned with positive probability. Suppose that a subject assigns probability a and $1 - a$ to the intervals $[x, y)$ and $[y, z)$ respectively, where the intervals have possibly different width. Denote with b and $1 - b$ the probability mass that interval $[x, y)$ and $[y, z)$ respectively would have, were the probability mass of each interval proportional to

[25] Notice that in 33 out of 329 observations the intervals are not all adjacent to each other.

their width. Therefore $b = \frac{y-x}{z-x}$ and $1 - b = \frac{z-y}{z-x}$. In the case of intervals of equal width, $b = 1/2$. We assume that the subjective distribution has the shape of an isosceles triangle whose base includes all of the interval with a probability mass more than proportional to its width and part of the other interval. If $a < b$ (i.e., interval $[x, y)$ has a probability mass less than proportional to its width), then we assume that the subjective distribution has the shape of an isosceles triangle whose base includes all of the interval $[y, z)$ and part of the interval $[x, y)$. Letting $t = \frac{(z-y)\sqrt{\frac{a}{2}}}{1-\sqrt{\frac{a}{2}})}$, it is straightforward to show that the isosceles triangle with height $h = \frac{2}{z-y+t}$ and endpoints $y - t$ and z defines a subjective probability density function that is consistent with the subject's reported beliefs.[26] This procedure generalizes Engelberg, Manski and Williams (2009) to the case of positive probability being assigned to two adjacent intervals that have unequal width.[27]

Finally, if the two intervals assigned with positive probability are non-adjacent, then fitting a Beta distribution is not possible and assuming that the subjective distribution has the shape of a isosceles triangle ranging over both intervals does not seem reasonable. This occurs in 20 out of 456 observations (4%).[28] We decide to exclude these observations from the analysis.

Table 13: Sample frequency of the number of intervals assigned with a positive probability. For each number of intervals between 2 and 5, the percentage of observations in which intervals are adjacent to one another.

Number of intervals assigned with positive probability	**non adjacent intervals** No.	%	**adjacent intervals** No.	%	**all** No.	%
1					43	100%
2	20	24%	64	76%	84	100%
3	17	34%	33	66%	50	100%
4	13	11%	107	89%	120	100%
5	3	10%	28	90%	31	100%
6					128	100%

When fitting a Beta distribution, the CDF defined over $[L, R]$ and evaluated at x is denoted $Beta(x, \alpha, \beta, L, R)$, where α and β are the shape parameters and L and R are the location parameters. We put no constraint on the value of α and β, thus allowing for unimodal, uniform, U-shaped, strictly increasing or strictly decreasing distributions. For each subject i and period t, we use the elicited $Q_{i,t}(r_j)$ to find the parameters $\alpha_{i,t}$ and $\beta_{i,t}$ that solve the least-squares problem:

[26] If instead $a > b$ (i.e., interval $[x, y)$ has a probability mass more than proportional to its width), then we assume that the subjective distribution has the shape of an isosceles triangle whose base includes all of the interval $[x, y)$ and part of the interval $[y, z)$. Letting $t = \frac{(y-x)\sqrt{\frac{1-a}{2}}}{1-\sqrt{\frac{1-a}{2}})}$, it is straightforward to show that the isosceles triangle with height $h = \frac{2}{y-x+t}$ and endpoints x and $y + t$ defines a subjective probability density function that is consistent with the subject's reported beliefs.

[27] Out of the 64 observations with two adjacent intervals, only 6 observations have two intervals with unequal width. This case is not present in the dataset analyzed by Engelberg, Manski and Williams (2009), where all (bounded) intervals have equal width.

[28] This case is not present in the dataset analyzed by Engelberg, Manski and Williams (2009).

$$\min_{\alpha,\beta} \left\{ \sum_{j=1}^{6} [Beta(r_j, \alpha_{i,t}, \beta_{i,t}, L_{i,t}, R_{i,t}) - Q_{i,t}(r_j)]^2 \right\}. \quad (8)$$

There is inevitably some arbitrariness in using a specific criterion to fit the experimental subjects' answers to a distribution, but the obtained beta CDFs fit the answers well: the 25th, 50th and 75th percentiles of the sample distribution of the minimized objective function are 0, 0.0013 and 0.0045, respectively. The sample average is 0.0052 and the largest value is 0.0654.

Table 14 reports summary statistics of the empirical distribution of the absolute differences $|A_{i,t} - M_{i,t}|$ and $|A_{i,t} - Me_{i,t}|$, distinguishing between the cases in which a Beta distribution or a triangle distribution is fitted to the data. As mentioned above, we compute the fitted mean M and the fitted median Me for all observations except those in which positive probability is assigned to two non-adjacent intervals.[29] The empirical distributions of both $|A_{i,t} - M_{i,t}|$ and $|A_{i,t} - Me_{i,t}|$ have 25th, 50th and 75th percentiles equal to 0, 5 and 15 percent respectively.[30] Therefore, both the fitted mean and the fitted median seem to provide a good proxy for the elicited *most likely value*.

One observation that was made under nonparametric analysis holds in an analogous way under parametric analysis too. As table 15 reports, the treatments in which subjects are asked to report their second-order beliefs as their first task, labeled 21C and 2C1, are associated with a significantly larger absolute difference between the elicited $A_{i,t}$ and the fitted mean $M_{i,t}$ and a significantly larger absolute difference between the elicited $A_{i,t}$ and the fitted median $Me_{i,t}$, compared to all other treatments. A Wilcoxon-Mann-Whitney test rejects at the 1 percent level (z=3.197, p=0.0014) the hypothesis that the distribution of $|A_{i,t} - M_{i,t}|$ is the same in treatments '21C and 2C1' and in all other treatments. The hypothesis that the distribution of $|A_{i,t} - Me_{i,t}|$ is the same in treatments '21C and 2C1' and in all other treatments is rejected at the 0.1 percent level (z=3.892, p=0.0001).[31]

Conclusion

Using the answers reported by subjects in terms of probabilities over intervals, we compute nonparametric bounds on the the subjective mean and the subjective median. Using the same answers, we also fit a parametric distribution and determine the mean and the median of the fitted subjective distribution. Thus, we have both nonparametric and parametric information about the subjective distribution and we can answer the following question: are the answer in terms of 'most likely value' and the answer in terms of probabilities over intervals coherent? In other words, are subjects reporting their probabilistic second-order beliefs coherently?

Using nonparametric methods, we find that the elicited 'most likely value' lies within the bounds on the mean and the median of the subjective distribution between 70% and 80% of the time. Using parametric fitting, we find that the elicited 'most likely value' matches very closely both the mean and the median of the fitted subjective distribution. Thus, evidence from both nonparametric and

[29] For the Beta distribution, the fitted mean is $M = \frac{\alpha}{\alpha+\beta}$. For the fitted median there is no close form solution.

[30] The empirical distributions of both $A_{i,t} - M_{i,t}$ and $A_{i,t} - Me_{i,t}$ have 25th, 50th and 75th percentiles equal to -4, 0 and 7 percent respectively.

[31] Analogous tests also reject (i) that the distribution of $|A_{i,t} - M_{i,t}|$ in treatments '21C and 2C1' has the same median than in all other treatments and (ii) that the distribution of $|A_{i,t} - Me_{i,t}|$ has the same median in treatments '21C and 2C1' than in all other treatments For (i) chi2=4.444 , p=0.035 and for (ii) chi2=12.496, p=0.000.

Table 14: Sample distribution (by fitting method) of the absolute difference between the elicited *most likely value* and the fitted mean or median of the subjective second-order expectations. Percentage points.

	|'most likely value'-fitted mean|				
	mean	25th perc.	50th perc.	75th perc.	obs.
Beta distribution	11	0	7	16	329
of which					
unimodal ($\alpha > 1, \beta > 1$)	8	0	5	12	155
uniform ($\alpha = 1, \beta = 1$)	8	0	0	12	29
U-shaped ($\alpha < 1, \beta < 1$)	14	0	7	23	108
strictly decreasing ($\alpha < 1, \beta > 1$)	15	9	16	20	11
strictly increasing ($\alpha > 1, \beta > 1$)	11	4	10	14	26
Triangle distribution	8	0	2	9	107
Beta and Triangle distr.	10	0	5	15	436
	|'most likely value'-fitted median|				
	mean	25th perc.	50th perc.	75th perc.	obs.
Beta distribution	11	0	8	16	329
of which					
unimodal ($\alpha > 1, \beta > 1$)	9	0	5	13	155
uniform ($\alpha = 1, \beta = 1$)	8	0	0	12	29
U-shaped ($\alpha < 1, \beta < 1$)	16	0	9	28	108
strictly decreasing ($\alpha < 1, \beta > 1$)	15	10	13	26	11
strictly increasing ($\alpha > 1, \beta > 1$)	11	6	11	17	26
Triangle distribution	8	0	2	9	107
Beta and Triangle distr.	10	0	5	15	436

Table 15: Sample distribution (by experimental treatment) of the absolute difference between the elicited *most likely value* and the fitted mean or median of the subjective second-order expectations. Percentage points.

	|'most likely value'-fitted mean|				|'most likely value'-fitted median|				
	mean	25th perc.	50th perc.	75th perc.	mean	25th perc.	50th perc.	75th perc.	obs.
Treatment									
12C	8	0	2	12	8	0	2	13	62
1C2	10	0	5	15	11	0	5	14	71
21C	15	4	10	24	16	5	12	25	68
2C1	11	0	4	15	12	0	7	16	102
C12	7	0	2	9	7	0	2	9	74
C21	8	0	7	15	9	0	5	13	59
all	10	0	5	15	10	0	5	15	436
Treatment									
21C and 2C1	13	1	7	18	13	2	8	20	170
other	8	0	4	13	9	0	4	13	266
all	10	0	5	15	10	0	5	15	436

parametric analysis suggests that the answers provided in probabilistic form by experimental subjects exhibit coherence. By conducting treatments that differ in the order in which subjects are asked to report choices, first- and second-order expectations, we examine whether task order has an impact on coherence. Both nonparametric and parametric analysis suggest that in those treatments, in which the second-order beliefs question is presented as first task, coherence of the answer in terms of 'most likely value' and the answer in terms of probabilities over intervals occurs less often than in the other treatments. In addition, we do not find that in those treatments, in which the second-order beliefs question is presented as first task, second-order beliefs are characterized by a distribution with a higher spread. In other words, we do not find evidence that subjects feel more uncertain about their second-order beliefs in those treatments compared to the others. The evidence of lower coherence without higher uncertainty may suggest that it is inherently more difficult for subjects to form their second-order beliefs before expressing their first-order beliefs and choices, independently of how uncertain those second-order beliefs may be.

Appendix B
Additional tables and figures

Table 16: Characteristics of the participants: field of studies and gender. Percentage sample frequency in each treatment.

		Treatment						
		12C	**1C2**	**21C**	**2C1**	**C12**	**C21**	**All treatments**
Field of Studies								
Humanities		19	28	22	35	20	31	26
Sciences		19	17	33	35	25	25	26
Social Sciences		57	50	45	27	50	44	44
	of which Economics	*38*	*22*	*39*	*23*	*45*	*25*	*32*
Undecided		6	6	0	4	5	0	4
Gender								
Female		62	78	61	38	55	75	60
Male		38	22	39	62	45	25	40

Table 17: Sample distribution of the subjective probability of opponent choosing A.

Subjective probability of opponent choosing A (in %)	**Period** 1 %	2 %	3 %	4 %	All %
0	9	10	12	7	9
10	0	0	1	0	0
15	0	1	0	0	0
20	1	1	3	3	2
25	4	4	2	3	3
30	4	4	4	5	4
35	1	1	0	0	0
40	9	6	11	4	7
42	0	0	1	0	0
45	2	1	1	0	1
48	0	1	0	0	0
49	0	0	0	1	0
50	44	52	46	45	46
52	0	0	0	1	0
55	0	1	1	2	1
58	0	0	0	1	0
60	7	7	8	4	7
61	0	1	0	0	0
65	1	1	0	0	0
70	1	1	1	4	2
75	3	1	3	2	2
80	2	1	2	4	2
85	1	0	0	0	0
90	0	0	0	2	0
95	1	0	0	0	0
100	11	9	7	15	11
All	100	100	100	100	100

Table 18: Sample distribution of subjective *most likely value* of opponent's beliefs of A being chosen.

Subjective most likely value of opponent's beliefs (in %)	**Period** 1 %	2 %	3 %	4 %	All %
0	6	2	4	7	5
1	1	0	0	0	0
10	0	0	1	0	0
20	0	3	3	4	2
25	0	0	0	1	0
30	4	2	3	4	3
35	0	0	1	2	1
40	4	8	2	3	4
42	0	0	0	1	0
45	0	1	2	0	1
50	50	53	55	59	54
52	0	0	1	0	0
55	0	1	0	1	0
60	5	9	9	7	7
65	1	2	2	0	1
67	0	1	0	0	0
70	4	4	5	4	4
72	1	0	1	0	0
75	6	4	3	2	4
80	1	2	1	1	1
90	1	0	1	1	1
100	16	11	9	4	10
All	100	100	100	100	100

Table 19: Sample distribution of the subjective cumulative distribution function, characterizing 2nd-order beliefs, evaluated at 0.5. Value of x such that $Q(0.5) = x$.

Subjective 2nd-order beliefs Q(0.5)=0.5 (in %)	**Period** **1** %	**2** %	**3** %	**4** %	**All** %
0	12	11	11	8	11
5	1	0	0	0	0
10	1	2	0	0	1
15	2	2	0	1	1
20	1	3	1	2	2
21	0	0	1	0	0
25	4	4	4	4	4
30	9	3	4	1	4
33	1	1	1	1	1
35	3	2	4	2	2
36	0	1	0	0	0
38	1	0	0	0	0
40	5	9	11	6	8
45	5	1	2	4	3
50	37	43	41	46	42
51	0	0	1	1	0
52	1	0	0	0	0
53	0	1	0	0	0
55	1	1	4	4	2
56	0	2	0	0	0
58	0	1	0	0	0
60	2	5	2	4	3
62	0	0	0	1	0
65	4	4	2	0	2
70	2	0	6	3	3
74	1	0	0	1	0
75	2	1	1	2	1
76	0	0	2	0	0
77	0	0	1	0	0
80	1	2	2	3	2
85	0	0	0	1	0
86	1	1	0	0	0
90	1	1	1	1	1
98	0	0	0	1	0
100	4	3	2	6	4
All	100	100	100	100	100

Table 20: Sample distribution of field of studies and gender for different values of the subjective probability of opponent choosing A.

Subj. prob. of opponent choosing A (in %)		Field of Studies: Humanities	Sciences	Social Sciences	Undecided	Gender: Female	Male	All
[0,5]	obs.	17	12	12	2	20	23	43
		14.17 *	**10**	**6**	*12.5*	**7.35**	**12.5**	*9.43*
(5,20]	obs.	5	2	3	0	6	4	10
	%	*4.17*	*1.67*	*1.5*	*0*	*2.21*	*2.17*	*2.19*
(20,50)	obs.	12	29	34	0	46	29	75
	%	*10*	*24.17*	*17*	*0*	*16.91*	*15.76*	*16.45*
50	obs.	45	46	115	6	139	73	212
		37.5 ***	**38.33** ***	**57.5**	*37.5*	**51.1**	**39.67** *	*46.49*
(50,80]	obs.	24	15	19	6	39	25	64
	%	*20*	*12.5*	*9.5*	*37.5*	*14.34*	*13.59*	*14.04*
(80,95]	obs.	1	0	2	1	1	3	4
	%	*0.83*	*0*	*1*	*6.25*	*0.37*	*1.63*	*0.88*
(95,100]	obs.	16	16	15	1	21	27	48
	%	**13.33**	**13.33**	**7.5**	*6.25*	**7.72**	**14.67** *	*10.53*
All	obs.	120	120	200	16	272	184	456
	%	*100*	*100*	*100*	*100*	*100*	*100*	*100*

Note: Benchmark category in Field of Studies is Social Sciences. Benchmark category in Gender is Female. *** denotes significantly different from the benchmark category at 0.1 percent, ** denotes significantly different from the benchmark category at 1 percent, * denotes significantly different from the benchmark category at 5 percent.

Table 21: Sample distribution of field of studies and gender for different values of the subjective *most likely value* of opponent's beliefs of A being chosen.

		Field of Studies				Gender		
Subj. most likely value of opponent's beliefs		**Humanities**	**Sciences**	**Social Sciences**	**Undecided**	**Female**	**Male**	**All**
[0,5]	obs.	13	4	4	1	14	8	22
	%	**10.83** ***	**3.33**	**2**	6.25	**5.15**	**4.35**	4.82
(5,20]	obs.	3	4	3	1	6	5	11
	%	2.5	3.33	1.5	6.25	2.21	2.72	2.41
(20,50)	obs.	12	13	17	0	26	16	42
	%	10	10.83	8.5	0	9.56	8.7	9.21
50	obs.	56	67	117	7	162	85	247
	%	**46.67** *	**55.83**	**58.5**	43.75	**59.56**	**46.2** **	54.17
(50,80]	obs.	16	26	40	4	49	37	86
	%	13.33	21.67	20	25	18.01	20.11	18.86
(80,95]	obs.	1	0	0	2	1	2	3
	%	0.83	0	0	12.5	0.37	1.09	0.66
(95,100]	obs.	19	6	19	1	14	31	45
	%	**15.83**	**5**	**9.5**	6.25	**5.15**	**16.85** ***	9.87
All	obs.	120	120	200	16	272	184	456
	%	100	100	100	100	100	100	100

Note: Benchmark category in Field of Studies is Social Sciences. Benchmark category in Gender is Female. *** denotes significantly different from the benchmark category at 0.1 percent, ** denotes significantly different from the benchmark category at 1 percent, * denotes significantly different from the benchmark category at 5 percent.

Appendix C
Instructions

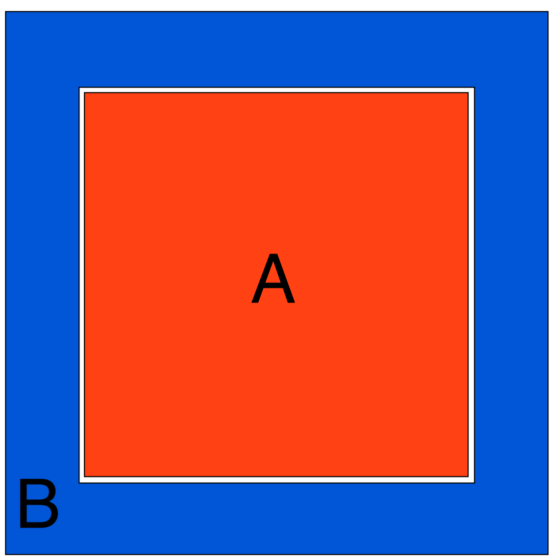
Welcome! Thank you for participating in this experiment. You are going to take part in a study of decision making. Please follow these instructions carefully. You will be paid according to your performance. In case you have questions, please raise your hand at any time. Please do not speak to other participants during the experiment.

You are going to be randomly paired with another student in the room to play a hide-and-seek game. In the hide-and-seek game, one person chooses a place where to hide a prize, and the other person needs to predict where the prize is hidden. The prize is a $10 banknote.

The prize must be hidden somewhere in a field. The field is divided into two zones: an inner zone and an outer zone. The two zones are identical in area. The figure below represents the field. In the figure, the inner zone is labelled A and colored in red and the outer zone is labelled B and colored in blue.

The hider has to choose between hiding the prize in zone A or in zone B. The seeker has to choose between predicting that the prize is hidden in zone A or in zone B. If the seeker does not predict correctly, then the hider wins the $10 prize. If instead the seeker predicts correctly, then he or she wins the $10 prize.

You will play the game for four rounds. In each round you will be randomly paired with another student. Therefore, you will not necessarily play the game every time with the same person. In each round the roles of hider and seeker will be assigned randomly to you and the other student who is paired with you. You will see the information on the screen. At the end of the session, the computer will randomly select one of the rounds, and you will be paid according to your performance in that round only.



Additional tasks

Besides having the opportunity to earn the $10 prize, you will also be given the opportunity to earn extra money by making forecasts. You will be asked to forecast the choice made by your opponent. You may be asked to assign a percent chance to each possible outcome and/or you may be asked to specify what you think the most likely outcome is.

A percent chance is a number between 0 and 100 percent, where 100 percent chance assigned to an outcome means that you are certain that such outcome is going to be the correct one, and 0 percent chance means that you are certain that such outcome is *not* going to be the correct one.

You will be paid based on the accuracy of your forecasts.

When forecasts are expressed in terms of what you think the most likely outcome is, you will earn nothing is your forecast is wrong (i.e., if the correct answer does not coincide with the one you chose), while you will earn $2 if your forecast is correct (i.e., if the correct answer coincides with the one you chose).

When forecasts are expressed in terms of a percent chance, then the payoff is determined as follows. Suppose that you are asked to assign a percent chance to two possible outcomes. For convenience let's label the two outcomes X and Y. Suppose that you assign percent chance p_X to outcome X and percent chance p_Y to outcome Y. We will give you $2 from which we will subtract an amount which depends on how inaccurate your answer was.

If outcome X turns out to be the correct one, the amount $(1 - \frac{p_X}{100})^2 + (\frac{p_Y}{100})^2$ is subtracted from the initial $2.

If outcome Y turns out to be the correct one, the amount $(\frac{p_X}{100})^2 + (1 - \frac{p_Y}{100})^2$ is subtracted from the initial $2.

Lets consider an example. Suppose that the correct outcome is X. Then **the worst you can do** is to assign a 0 percent chance to X and a 100 percent chance to Y. In this case your payoff is:

$$\$2 - \$\left(1 - \frac{0}{100}\right)^2 - \$\left(\frac{100}{100}\right)^2 = \$2 - \$(1)^2 - \$(1-0)^2 = \$2 - \$1 - \$1 = \$0$$

The best you can do is instead to assign a 100 percent chance to X and 0 percent chance to Y. In this case your payoff is:

$$\$2 - \$\left(1 - \frac{100}{100}\right)^2 - \$\left(\frac{0}{100}\right)^2 = \$2 - (1-1)^2 - (0)^2 = \$2 - \$0 - \$0 = \$2$$

Therefore your payoff in this task is between $0 and $2.

The same rule is applied when more than two possible outcomes exist. Your payoff in this task is always between $0 and $2.

Note Since your forecasts are made when you don't know what your opponent has chosen, **the best thing you can do to maximize the expected size of your payoff is to simply state what you think.**

Final comments

By participating to the game you will receive a show-up fee of $5, plus the $10 prize if you are the winner in the game, plus payment based on the accuracy of your forecasts. Payment for the forecasts ranges between $0 and $6.

Therefore, the total payment you will receive will be between $5 and $21.