Does a Structural Macroeconomic Model Help Long-Term Portfolio Management?

Ludovic Calés[a], Eric Jondeau[b], Michael Rockinger[c]

August 2016

Abstract

This paper considers an institutional investor with a dynamic long-term portfolio allocation perspective. We compare the performance of two competing forecasting models: an unrestricted VAR model and a fully-structural Dynamic Stochastic General Equilibrium (DSGE) model to describe the dynamics of the U.S. economy and financial markets. The models include financial variables such as the financial market values, dividend payments, and long-term government bond returns. We find that the DSGE model outperforms the unrestricted VAR model in forecasting financial returns in the long term. Even if it has much less unknown parameters, the DSGE model benefits from economically-grounded realistic values. Finally, the DSGE model also outperforms the VAR model in a long.term portfolio allocation exercise.

Keywords: Long-term portfolio management, VAR model, DSGE model, Asset return forecasting.

JEL Classification: C11, E44, G11.

[a]Faculty of Business and Economics (HEC Lausanne), University of Lausanne, CH 1015 Lausanne, Switzerland. E-mail: Ludovic.Cales@unil.ch.

[a]Swiss Finance Institute and Faculty of Business and Economics (HEC Lausanne), University of Lausanne, CH 1015 Lausanne, Switzerland. E-mail: Eric.Jondeau@unil.ch. Corresponding author.

[b]Swiss Finance Institute and Faculty of Business and Economics (HEC Lausanne), University of Lausanne, CH 1015 Lausanne, Switzerland. E-mail: Michael.Rockinger@unil.ch.

We benefited from comments and discussions with Martin Boons and Didier Maillard. The usual disclaimer applies.

1 Introduction

Given the population's increasing life span, institutional investors (such as insurance companies and pension funds) have an increasing social responsibility to allocate their funds in an optimal manner. The allocation must be made on a long-term basis and the institutional investor must account for the expected evolution of financial markets. The theoretical literature provides some guidance on how such allocations may be performed. If asset returns are independently and identically distributed (i.i.d) and investor preferences do not change over time, then a simple buy-and-hold strategy is optimal and the optimal portfolio weights are the same across different investment horizons (Samuelson, 1969). Merton (1969, 1971) discusses multiperiod portfolio allocation under the assumption that the distribution of returns fluctuates over time. In such a case, it is optimal for the investor to hedge against adverse movements of expected returns. In addition, if the investment opportunity set depends on certain state variables, the investor should invest in a hedging portfolio based on these state variables, and thus, the long-term portfolio will differ from a buy-and-hold portfolio.

The long-term predictability of financial returns plays a major role in the context of portfolio allocation. Brennan et al. (1997) empirically analyze the impact of myopic versus dynamic portfolio choices. The authors find that, because of mean reversion in stock and bond returns, an investor with a long horizon will place a larger fraction of her wealth in stocks and bonds compared to an investor with a short horizon. Barberis (2000) reports that, even after parameter uncertainty is taken into account, sufficient stock return predictability remains, leading investors to include more stocks in their portfolios. Campbell and Viceira (1999, 2001, 2002) and Campbell et al. (2003) investigate several aspects of long-term investment in a Vector AutoRegressive (VAR) model with predictable asset returns. They observe that a long-term investor adopting a dynamic portfolio strategy should prefer stocks to long-term bonds because, in their model, the intertemporal hedging demand is positive for stocks and negative for nominal long-term bonds.[1]

[1]Long-term portfolio allocation has also been investigated in the context of factor models. Sangvinatsos and Wachter (2005) use an affine term structure model to investigate bond return predictability. They find that allowing for a time-varying bond risk premium results in a high hedging demand for

Several papers demonstrate that macro variables are useful for predicting financial returns. See, e.g., Cochrane (1991), Lettau and Ludvigson (2001), Santos and Veronesi (2006), and Cooper and Priestley (2008) for stock returns and Ang and Piazzesi (2003), Hördahl et al. (2008), and Rudebusch and Swanson (2008) for bond returns. Most of the empirical work building on long-term predictions of financial returns has used reduced-form models for forecasting financial returns. For instance, all the papers mentioned above rely on a VAR(1) model or a factor model to estimate the dynamics of financial returns. These specifications are appealing approaches because long-term forecasting can be performed in a straightforward manner. However, parameters are not structural (or deep) under this approach, and thus, they are likely to be affected by changes in government and monetary decisions, as highlighted by Lucas (1976). These models also raise the issue of non-stationarity. Several key variables in these models, such as the short-term rate or the dividend-price ratio, have near-to-unit-root dynamics.

In this paper, we study the optimal dynamic allocation for a long-term institutional investor, adopting the approach of Campbell et al. (2003). However, we do not only use a VAR model to predict financial returns but we also consider a structural Dynamic Stochastic General Equilibrium (DSGE) model to address the issues mentioned above. We build on the model described by Smets and Wouters (2003, 2007) and extended by Alpanda (2013), which allows for interactions between the business cycle and the stock market. A well-known limitation of DSGE macro-finance models is that, once log-linearized, they only generate constant risk premia. This problem has been addressed in several papers, following Rudebusch et al. (2007), by investigating alternative ways of introducing time variability in the risk premia. We follow the approach proposed by Marzo et al. (2008) and Falagiarda and Massimiliano (2012), who describe the bond market segmentation through portfolio adjustment frictions (or rebalancing costs), which give rise to dynamic bond and stock risk premia in the log-linearized model. Risk premia are allowed to depend on the various structural shocks that affect the economy. This

long-term bonds. Koijen et al. (2010) use a model with a factor structure to evaluate the importance of the bond premium for a life-cycle investor with short sales and borrowing constraints. They find that, with a time-varying bond risk premium, the hedging demand is negative for stocks and positive for bonds, although the magnitude of the effects is limited by the restrictions on short selling.

approach provides the DSGE model with some flexibility in the evolution of risk premia through time.

We estimate the VAR and DSGE models based on U.S. data over a period of 60 years, from 1955 to 2014. Given our focus on long-term investment, we evaluate the ability of these models to predict the evolution of financial returns in the long term. We first investigate the term structure of risks of financial assets across investment horizons. We find that, in the DSGE model, the annualized volatility of stock returns decreases with the investment horizon, consistent with the mean reversion found in stock returns. The annualized volatility of long-term bond return also decreases with the horizon, although to a lesser extent, and hence stocks are relatively less risky than bonds in the long term. This feature is not shared by the VAR model, which does not generate mean reversion in stock returns in our sample. These patterns are highly stable over time. Second, although both types of models have a similar forecasting ability for macro variables, the DSGE model strongly outperforms the VAR model in forecasting financial returns in the long term. This result is likely due to the long-term economic restrictions imposed by the DSGE model, which help constraining long-term financial returns to take realistic values.

Regarding the optimal allocation exercise, the DSGE model generates a high positive hedging demand for stock. This finding explains why an investor with low risk aversion holds large fractions of her wealth in stocks. In contrast, the VAR model generates a negative hedging demand for stocks, resulting in a portfolio with limited stock holding.

Finally, we evaluate the out-of-sample performances of dynamic strategies based on the VAR and DSGE models. As stocks and long-term bonds are very good hedges against changes in the stock and bond premia in the DSGE model, they are associated with high positive hedging demands. As a result, the DSGE portfolio is typically long bonds and stocks and short cash. In contrast, long-term bonds do not help with hedging the bond premium in the VAR model, so the VAR portfolio typically comprises long stocks and cash and short bonds. In addition, for all levels of risk aversion and long investment horizons, the DSGE model exhibits higher expected returns and Sharpe ratios than the

VAR model. We interpret this result to occur because of the better ability of the DSGE model to describe the dynamics of the stock and bond premia.

The remainder of the paper is organized as follows. In Section 2, we describe how a long-term investor determines its optimal allocation in a long-run perspective. In Section 3, we present the VAR and DSGE models that we use to forecast future financial returns. In Section 4, we compare the out-of-sample performances of long-term investors using a VAR or a DSGE model to forecast financial returns. The final comments are presented in Section 5.

2 Optimal Dynamic Strategy

We consider the optimal dynamic investment policy of a long-term institutional investor who uses a macro-finance model to predict future asset returns. Dynamic strategies take into account changes in investment opportunities, such that the portfolio is, in principle, rebalanced in an optimal manner at specified regular intervals. Because the investor wishes to hedge the portfolio against adverse changes in the investment set, the strategy gives rise to intertemporal hedging demands (Merton, 1973).

One difficulty encountered with dynamic strategies is that, generally, no closed-form solution exists. See Kim and Omberg (1996) and Wachter (2002) for exact analytical solutions to continuous-time intertemporal portfolio-choice problems and Bodie et al. (2009) and Wachter (2010) for recent surveys on dynamic allocation. In discrete time, some solutions to this problem based on numerical techniques (Barberis, 2000; Lynch, 2001) or analytical approximations of the solution (Campbell and Viceira, 1999, 2001, 2002 and Campbell et al., 2003) have been proposed. For instance, Barberis (2000) considers the case of a single asset with no intermediate consumption and simulates the path of the state variables over the investment horizon by using a discretization scheme.

We now describe our framework, which follows Campbell et al. (2003) and is well suited for evaluating dynamic investment strategies. We consider an institutional investor, for instance, an insurance company, a pension fund, a sovereign wealth fund, an endowment, or a charity, with a long-term horizon and regular payouts to be made. The investor has

Epstein and Zin (1989) recursive preferences described as:

$$U(O_t, E_t[U_{t+1}]) = \left[(1-\beta)O_t^{(1-\gamma)/\theta} + \beta(E_t[U_{t+1}^{1-\gamma}])^{1/\theta}\right]^{\theta/(1-\gamma)},$$

where O_t as the net outflow that has to be made by the investor in period t, $\gamma > 0$ is the relative risk aversion coefficient, $\psi > 0$ is the elasticity of intertemporal substitution, and $\theta = (1-\gamma)/(1-\psi^{-1})$.[2]

We consider three asset classes: cash, bonds, and stocks. Investment in bonds is based on a rolled strategy in constant-maturity bonds: the bond portfolio is rebalanced in each period to keep the maturity constant over time. The investor buys a 10-year bond at the beginning of the quarter and sells it at the end of the quarter. Then, at the beginning of the next quarter, she buys a new 10-year bond for one quarter. The bond position is, therefore, assumed to be rolled over time with quarterly rebalancing. We denote by $R_{1,t+1}$ is the gross real one-period interest rate, $R_{b,t+1}$ is the gross real holding-period return for a 10-year bond, and $R_{s,t+1}$ is the gross real stock return.

The intertemporal budget constraint is:

$$W_{t+1} = (W_t - O_t)R_{p,t+1},$$

where $R_{p,t+1} = R_{1,t+1} + \alpha_{b,t}(R_{b,t+1} - R_{1,t+1}) + \alpha_{s,t}(R_{s,t+1} - R_{1,t+1})$ is the real portfolio return, with $\alpha_{b,t}$ and $\alpha_{s,t}$ denoting the fraction of wealth invested in bonds and stocks, respectively.[3]

Although the models used for forecasting are described in Section 3, we now briefly summarize how forecasts of financial returns are obtained. We define $z_{t+1} = (r_{b,t+1} - r_{1,t+1}, r_{s,t+1} - r_{1,t+1})$ the vector of log excess returns with $r_{i,t+1} = \log(R_{i,t+1})$, for $i = 1, b, s$. Forecasts are based on a set of state variables, which we denote by s_{t+1} and we describe in the next section. The state variables include all the variables in a given (VAR or DSGE) model. Their dynamics is given by $s_{t+1} = \mu + G\ s_t + H\ \eta_{t+1}$, where η_{t+1} is the vector

[2]Epstein-Zin utility simplifies to the power utility when $\gamma = \psi^{-1}$ and to the log-utility when $\gamma = \psi^{-1} = 1$.

[3]The one-period return $R_{1,t+1}$ is not risk free at long horizons, but, for convenience, it is still used to define excess returns in this expression.

of innovations. We assume that η_t is normally distributed with mean 0 and covariance matrix Σ_η. Then the vector of log excess returns is obtained by $z_{t+1} = \Phi_0 + \Phi_1 s_{t+1}$. The vector of parameters $\Phi_0 = \Phi_1 \mu$ contains the long-term value of the observables. Matrix Φ_1 is simply a selection matrix because the log excess returns are (combinations of) state variables.

In this framework, Campbell and Viceira (1999, 2001, 2002) and Campbell et al. (2003) provide an approximate analytical solution based on the log-linear approximation of the Euler equation and the intertemporal budget constraint. The dynamic optimization problem is solved and the optimal portfolio and outflow are shown to be linear in the state variables:

$$\alpha_t = A_0 + A_1 s_t, \tag{1}$$

$$o_t - w_t = b_0 + B_1' s_t + s_t' B_2 s_t, \tag{2}$$

with $o_t = \log(O_t)$ and $w_t = \log(W_t)$. The optimal portfolio rule α_t has two components: a myopic component, which corresponds to the one-period optimal allocation, and an intertemporal hedging portfolio, which accounts for movements in expected returns. Campbell et al. (2003) demonstrate that the coefficient matrices A_0 and A_1 for the optimal portfolio are:

$$\begin{aligned} A_0 &= \frac{1}{\gamma}\Sigma_{xx}^{-1}\left[\Phi_0 + \frac{1}{2}\sigma_x^2 + (1-\gamma)\sigma_{1x}\right] + \left(1 - \frac{1}{\gamma}\right)\Sigma_{xx}^{-1}\left(-\frac{\Lambda_0}{1-\psi}\right), \\ A_1 &= \frac{1}{\gamma}\Sigma_{xx}^{-1}\Phi_1 + \left(1 - \frac{1}{\gamma}\right)\Sigma_{xx}^{-1}\left(-\frac{\Lambda_1}{1-\psi}\right), \end{aligned}$$

where Σ_{xx} is the covariance matrix of excess returns, $\sigma_x^2 = \text{diag}(\Sigma_{xx})$ is the vector of variances of excess returns, and σ_{1x} is the covariance between the one-period rate and the excess returns. Λ_0, and Λ_1 are related to the covariance of the assets and the outflow-wealth ratio and therefore to the model's parameters. These parameters and the optimal consumption policy (b_0, B_1, and B_2) are precisely defined in the appendix of Campbell et al. (2003).

The first terms in A_0 and A_1 correspond to the myopic component, which is the solution to the mean-variance allocation problem when excess returns are predictable. The first term of A_1 takes the prediction of excess returns into account as long as $\Phi_1 \neq 0$. The second terms in A_0 and A_1 correspond to the intertemporal hedging demands. They appear in the optimal portfolio weights as long as $\gamma \neq 1$ and depend on the ability of the assets to hedge the investor against a deterioration of outflow. When $\gamma = \psi = 1$ (log-utility), the hedging demands vanish.[4]

Once the optimal portfolio plan (A_0 and A_1) is determined, the investor allocates her wealth every period according to the last realization of the state variables. Assume that the sample period $(1, \cdots, T_0)$ is used to estimate the model and determine the optimal plan. Then portfolio weights α_t are determined every period once state variables s_t are observed, for $t = T_0 + 1, \cdots, T$.

3 Forecasting Models

In this section, we describe the competing forecasting models, namely the VAR and DSGE models. We investigate two properties of these models: their ability to generate a realistic term structure of risks and to produce good forecasts of the real financial returns.

3.1 Data

We consider 10 observable variables: per-capita real GDP (GDP_t), per-capita real consumption ($CONS_t$), per-capita real investment (INV_t), per-capita labor hours (HRS_t), the real wage rate ($WAGE_t$), the GDP deflator (P_t), the per-capita real market value of nonfinancial firms (CAP_t), the per-capita real dividends of nonfinancial firms (DIV_t), the federal funds rate (FFR_t), and the 10-year Treasury bond interest rate (LTR_t). We transform the main variables of the model as follows: $y_t = \log(GDP_t)$, $c_t = \log(CONS_t)$,

[4] Campbell et al. (2003) show that ψ is essentially irrelevant for the determination of the optimal portfolio rule, although it directly drives the optimal outflow rule. In our experiments, we indeed observe that for a given value of γ, altering ψ does not affect the optimal allocation rule. In the empirical application, we follow Campbell et al. (2003) and focus on the case $\psi = 1$. In this case, the choice of the outflow o_t is myopic because the outflow-wealth ratio $o_t - w_t$ is constant, whereas the optimal portfolio rule is not myopic. The solution would be fully myopic in the log-utility case only.

$\iota_t = \log(INV_t)$, $h_t = \log(HRS_t)$, $wp_t = \log(WAGE_t)$, $\pi_t = \Delta\log(P_t)$, $vp_t = \log(CAP_t)$, $dp_t = \log(DIV_t)$, $r_{1,t} = \log(1 + FFR_t)$, $y_{b,t} = \log(1 + LTR_t)$. We also define the holding period return of the 10-year bond between t and $t+1$ as:

$$r_{b,t+1} = D_t \; y_{b,t} - (D_t - 1) \; y_{b,t+1}, \tag{3}$$

where D_t is Macaulay's duration.

The market value of nonfinancial firms is computed as the value of equity and dividends available from the aggregate balance sheet and flow of funds data. Following Alpanda (2013), the dividend series includes net buybacks, which allows us to take into account corporate finance issues in a more realistic framework. Instead of paying dividends, firms often prefer to buy back their own shares as a way of distributing cash to shareholders. Further details regarding the data are provided in Appendix A.

The sample comprises data from the 1955–2014 period (240 quarterly observations). **Figure 1** displays the data that are used to estimate the model, and **Table 1** provides basic statistics for the observable variables. As shown in the table, output growth mainly results from consumption, whereas investment growth is lower, on average, than consumption growth. By contrast, most of the output volatility is derived from real investment. Regarding asset returns, we note that the average inflation is lower than the average short-term rate and the average long-term bond interest rate (3.28% per year for inflation versus 5.13% for the short-term rate and 6.16% for the long-term rate). Finally, on average, the real excess bond return and the real excess stock return are 0.9% and 6.7% per year, respectively.

The figure also shows that most of the observable variables are clearly stationary, although hours (h_t), the dividend price ratio (dpr_t), the federal funds rate ($r_{1,t}^{(n)}$), and the 10-year yield to maturity ($y_{b,t}^{(n)}$) show high persistence. First-order autocorrelations are equal to 0.99, 0.917, 0.968, and 0.986, respectively.

[Insert Table 1 and Figure 1 here]

3.2 DSGE Model

This section briefly describes the DSGE model that is used in this paper. The model is based on the workhorse model developed by Christiano et al. (2005) and Smets and Wouters (2003, 2007) and incorporates some financial aspects proposed by Alpanda (2013). The model features price and wage stickiness, price and wage indexation, habit formation in consumption and investment adjustment costs, variable capital utilization and fixed costs in production. In this economy, monopolistically competitive intermediate good producers own capital stock and are price-setters in the goods market. Households own shares of the intermediate firms and are wage-setters in the labor market. Because stock returns depend on dividends and thus on a firm's earnings, we describe how the firm generates dividends using a model of the firm's revenues and expenses (wages, investment, and taxes), following Jermann (1998) and Boldrin et al. (2001). We further introduce Treasury bonds issued by the government as a way for households to transfer wealth from one period to another. DSGE models have been widely used to describe term structure dynamics because the pricing kernel can be implicitly derived from the macro model (Ang and Piazzesi, 2003; Wu, 2006; Hördahl et al., 2008; Bekaert et al., 2010).

An important issue in the construction of a macro-finance DSGE model is that, once log-linearized, the DSGE model does not to generate time-varying risk premia. This problem has been addressed in several papers, following Rudebusch et al. (2007), by investigating alternative ways of introducing time variability in the risk premia. For instance, Hördahl et al. (2008) and Rudebusch and Swanson (2008) consider higher-order approximations of the DSGE model; Van Binsbergen et al. (2012) and Rudebusch and Swanson (2012) introduce Epstein and Zin (1989) recursive preferences; Guvenen (2009) and De Graeve et al. (2010) allow for heterogeneous agents. In this paper, we follow the approach proposed by Marzo et al. (2008) and Falagiarda and Massimiliano (2012), who describe bond market segmentation through portfolio adjustment frictions. The justification for such frictions can be found in the "preferred habitat" theory (Modigliani and Sutch, 1966, 1967; Vayanos and Vila, 2009; Guibaud et al., 2008).[5] Given these frictions,

[5] Alpanda (2013) also introduces a time-varying risk premium for risky stocks; however, the shock affects the short-term bond holdings such that it cannot be extended to the case of several risky assets.

which we also interpret as rebalancing costs, households have a preference for holding bonds of different maturities, resulting in nonzero demands for the various maturities.

The complete description of the structural model is presented in Appendix B.1. The resulting log-linearized model is described as follows:

- Household's optimization program

  - FOC w.r.t. consumption

$$\sigma_C \ \hat{c}_{s,t} = (\sigma_C - 1) \ \xi \ \hat{h}_t - \hat{\lambda}_t + v_t,$$

    with consumption surplus $(1 - \zeta/\gamma) \ \hat{c}_{s,t} = \hat{c}_t - (\zeta/\gamma) \ \hat{c}_{t-1}$

  - FOC w.r.t. labor

$$\hat{c}_{s,t} + \sigma_L \ \hat{h}_t = \hat{wp}_t + \hat{\Omega}_{h,t}$$

  - FOC w.r.t. wages

$$\hat{\pi}_t^w = \eta_w \ \hat{\pi}_{t-1} + \tilde{\beta} \left(E_t \hat{\pi}_{t+1}^w - \eta_w \ \hat{\pi}_t \right) + \frac{1}{\kappa_w} \left(\hat{\Omega}_{h,t} + \hat{\psi}_t \right)$$

    with wage inflation $\hat{\pi}_t^w = \hat{wp}_t - \hat{wp}_{t-1} + \hat{\pi}_t$ and discount factor $\tilde{\beta} = \beta \eta \gamma^{1-\sigma_C}$

  - FOC w.r.t. short-term bonds

$$\hat{\lambda}_t = E_t \hat{\lambda}_{t+1} + (\hat{r}_{f,t}^{(n)} - E_t \hat{\pi}_{t+1})$$

  - Asset pricing condition for long-term bonds

$$\hat{\lambda}_t = E_t \hat{\lambda}_{t+1} + E_t \hat{r}_{b,t+1}^{(k)} - \hat{\phi}_{b,t}^{(k)}$$

See also Andrés et al. (2004) and De Graeve et al. (2010). Buss and Dumas (2012) propose an equilibrium model in which financial trade entails deadweight transaction costs. Abel and Blanchard (1983), Andrés et al. (2004), and Marzo et al. (2008) introduce a specific quadratic adjustment cost based on the change in the bond or stock holdings. As we describe in Appendix B.1.7, we remain agnostic on the specification of the adjustment cost and will specifically investigate its drivers. As households are atomistic, we do not expect the adjustment cost to be quadratic in holdings. In contrast, we expect the adjustment cost to vary with economic conditions.

– Asset pricing condition for shares

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + E_t\hat{r}_{s,t+1} - \hat{\phi}_{s,t}$$

with real stock return $\hat{r}_{s,t} = \tilde{\beta}\ \hat{vp}_t + (1-\tilde{\beta})\hat{dp}_t - \hat{vp}_{t-1}$

- Firm's optimization program

  – FOC w.r.t. capital

$$\hat{q}_t = \frac{\tilde{\beta}}{\eta\gamma} E_t\left[(1-\delta)\hat{q}_{t+1} + \frac{(1-\tau_y)(1-\tau_s)\alpha\chi}{\bar{k}/\bar{y}}\hat{u}_{t+1}\right] - \hat{\lambda}_t + E_t\hat{\lambda}_{t+1}$$

  – FOC w.r.t. labor

$$\hat{\Omega}_{f,t} + \frac{1}{\theta}\ \hat{y}_t = \hat{h}_t + \hat{wp}_t$$

  – FOC w.r.t. utilization rate

$$\hat{\Omega}_{f,t} + \frac{1}{\theta}\ \hat{y}_t = (1+\chi)\ \hat{u}_t + \hat{k}_{t-1}$$

  – FOC w.r.t. price

$$\hat{\pi}_t - \eta_p\ \hat{\pi}_{t-1} = \tilde{\beta}(E_t\hat{\pi}_{t+1} - \eta_p\ \hat{\pi}_t) + \frac{1}{\kappa_p}\left(\hat{\Omega}_{f,t} + \hat{\theta}_t\right)$$

  – FOC w.r.t. investment

$$\hat{\iota}_t - \hat{\iota}_{t-1} = \tilde{\beta}(E_t\hat{\iota}_{t+1} - \hat{\iota}_t) + \frac{1}{\kappa_I}(\hat{q}_t + \hat{z}_t^I)$$

  – Capital accumulation:

$$(\eta\gamma)\hat{k}_t = (1-\delta)\hat{k}_{t-1} + (\eta\gamma - 1 + \delta)\,(\hat{\iota}_t + \hat{z}_t^I)$$

– Production function:

$$\hat{y}_t = \theta \left[\hat{z}_t + \alpha(\hat{u}_t + \hat{k}_{t-1}) + (1-\alpha)\hat{h}_t \right]$$

– Real dividends:

$$\frac{\bar{dp}}{\bar{y}}\hat{dp}_t = (1-\tau_y)\left[(1-\tau_s)\hat{y}_t - (1-\alpha)(\hat{wp}_t + \hat{h}_t) \right] - \frac{\bar{\iota}}{\bar{y}}\hat{\iota}_t + \tau_y \delta_a \frac{\bar{k}}{\bar{y}}\hat{k}_{t-1} + \hat{\Phi}^d_t$$

- Other conditions

– Taylor rule of the central bank:

$$\hat{r}^{(n)}_{f,t} = \rho_r \ \hat{r}^{(n)}_{f,t-1} + (1-\rho_r)\left[a_\pi(\hat{\pi}_t - \hat{\bar{\pi}}_t) + a_y(\hat{y}_t - \hat{y}^n_t) + a_g \ \Delta(\hat{y}_t - \hat{y}^n_t) \right] + \hat{\epsilon}_t$$

with inflation target $\hat{\bar{\pi}}_t = \rho_{\bar{\pi}} \ \hat{\bar{\pi}}_{t-1} + \eta_{\bar{\pi},t} - \theta_{\bar{\pi}} \ \eta_{\bar{\pi},t-1}$

– Goods market clearing condition:

$$\frac{\bar{c}}{\bar{y}}\hat{c}_t + \frac{\bar{\iota}}{\bar{y}}\hat{\iota}_t + \frac{\bar{g}}{\bar{y}}\hat{g}_t = \hat{y}_t$$

Consistent with the approach promoted by Smets and Wouters (2003), Smets and Wouters (2007), and Alpanda (2013), the model is driven by several shocks: the shocks to the technology (z_t), the preferences (v_t), the wage markup (ψ_t), the price markup (θ_t), the investment-specific technology (z^I_t), the dividends (Φ^d_t), the government spending

(g_t), and the monetary policy (ϵ_t). All the shocks have an ARMA(1,1) dynamics:

$$
\begin{aligned}
\log(z_t) &= \rho_z \log(z_{t-1}) + \eta_{z,t} - \varsigma_z\ \eta_{z,t-1}, \\
\log(v_t) &= \rho_v \log(v_{t-1}) + \eta_{v,t} - \varsigma_v\ \eta_{v,t-1}, \\
\log(\psi_t) &= (1-\rho_\psi)\log(\psi) + \rho_\psi \log(\psi_{t-1}) + \eta_{\psi,t} - \varsigma_\psi\ \eta_{\psi,t-1}, \\
\log(\theta_t) &= (1-\rho_\theta)\log(\theta) + \rho_\theta \log(\theta_{t-1}) + \eta_{\theta,t} - \varsigma_\theta\ \eta_{\theta,t-1}, \\
\log(z_t^I) &= \rho_I \log(z_{t-1}^I) + \eta_{I,t} - \varsigma_I\ \eta_{I,t-1}, \\
\log(\Phi_t^d) &= \rho_d \log(\Phi_{t-1}^d) + \eta_{d,t} - \varsigma_d\ \eta_{d,t-1}, \\
\log(g_t) &= (1-\rho_g)\log(g) + \rho_g \log(g_{t-1}) + \rho_{g,z}\ \eta_{z,t} + \eta_{g,t} - \varsigma_g\ \eta_{g,t-1}, \\
\log(\epsilon_t) &= \rho_r \log(\epsilon_{t-1}) + \eta_{r,t} - \varsigma_r\ \eta_{r,t-1},
\end{aligned}
$$

where η denotes mutually uncorrelated i.i.d. innovations. We also allow the technology innovation to affect the government spending shock.

Risk premia on bonds and stocks are defined as:

$$
\begin{aligned}
\phi_{b,t}^{(k)} &= -E_t[\log(1+\Phi_{b,t+1}^{(k-1)}) - \log(1+\Phi_{b,t}^{(k)})], \\
\phi_{s,t} &= -E_t[\log(1+\Phi_{s,t+1}) - \log(1+\Phi_{s,t})].
\end{aligned}
$$

We adopt an agnostic view for their modeling. We allow risk premia to have an ARMA(1,1) structure and to depend on all the other shocks of the economy:

$$
\begin{aligned}
\begin{bmatrix} \phi_{b,t}^{(k)} \\ \phi_{s,t} \end{bmatrix} &= \begin{bmatrix} \rho_b^{(k)} & 0 \\ 0 & \rho_s \end{bmatrix} \begin{bmatrix} \phi_{b,t-1}^{(k)} \\ \phi_{s,t-1} \end{bmatrix} + \begin{bmatrix} 1 & \rho_b^{(k)} \\ \rho_s & 1 \end{bmatrix} \begin{bmatrix} \eta_{b,t}^{(k)} \\ \eta_{s,t} \end{bmatrix} \\
&+ \begin{bmatrix} \varsigma_b^{(k)} & 0 \\ 0 & \varsigma_s \end{bmatrix} \begin{bmatrix} \eta_{b,t-1}^{(k)} \\ \eta_{s,t-1} \end{bmatrix} + \begin{bmatrix} \tilde{\rho}_b^{(k)\prime} \\ \tilde{\rho}_s^{\prime} \end{bmatrix} \eta_{S,t},
\end{aligned}
$$

where $\eta_{S,t} = (\eta_{z,t}, \eta_{v,t}, \eta_{\psi,t}, \eta_{\theta,t}, \eta_{I,t}, \eta_{d,t}, \eta_{g,t}, \eta_{r,t})'$. These dynamics of the risk premia are expected to capture the second-order properties of the DSGE model that are lost in the log-linearization of the model. See Hördahl et al. (2008).

We also consider a restricted specification of the DSGE model, in which the risk premia are pure ARMA(1,1) processes:

$$
\begin{aligned}
\phi_{b,t}^{(k)} &= \rho_b^{(k)}\ \phi_{b,t-1}^{(k)} + \eta_{b,t}^{(k)} - \varsigma_b^{(k)}\ \eta_{b,t-1}^{(k)}, \\
\phi_{s,t} &= \rho_s\ \phi_{s,t-1} + \eta_{s,t} - \varsigma_s\ \eta_{s,t-1}.
\end{aligned}
$$

This restricted model allows us to measure the contribution of the other shocks to the dynamics of the risk premia.

The model has 10 observable variables $x_t^{(DSGE)} = (\Delta y_t, \Delta c_t, \Delta \iota_t, h_t, \Delta wp_t, \pi_t, r_{1,t}^{(n)}, y_{b,t}^{(n)}, \Delta vp_t, \Delta dp_t)'$. We notice that the real stock return is not an observable variable of the model, following Alpanda (2013). Instead, it is deduced from the firm's valuation through the dividend discount model. The vector of state variables, denoted by $s_t^{(DSGE)}$, includes the (demeaned) observables, $x_t^{(DSGE)}$, the shocks, and the future expected variables. The model can be written as a backward-looking state-space representation, which is akin to a restricted VAR(1) model:

$$
s_{t+1}^{(DSGE)} = \mu_D + G_D\ s_t^{(DSGE)} + H\ \eta_{t+1}^{(DSGE)}, \tag{4}
$$

where $\eta_t^{(DSGE)}$ is the set of all the structural innovations.

In the full DSGE model, there are 67 parameters: 15 structural parameters, 24 parameters for the dynamics of the risk premia, and 28 for the dynamics of the other shocks. The restricted DSGE model has 49 parameters. Parameters are estimated using Bayesian methodology. The estimation technique is described in Appendix B.2 and the parameter estimates are reported in Tables A1 to A3. As for all macro-finance models, the estimation of the DSGE model over a long sample period is potentially plagued by two issues, nonstationarity and parameter instability. Some of the autoregressive parameters are found to be large, suggesting highly persistent dynamics of the shocks. Such is the case in particular for the technology shock and the government shock, which clearly reflect near-to-unit-root behavior. We also investigate the stability of the parameter estimates over time by reestimating the model with a rolling window. Parameter dynamics are reported in the Technical Appendix. We find that most of the parameters are relatively stable over time, while the dynamics of a few parameters (such as the adjustment cost of prices or the inflation parameter in the reaction function) have a trend. This issue is partly due to the Federal Reserve's adoption of a nonborrowed reserve operating procedure during the 1979–1982 period.

The variability of the parameters over time indicates that the forecasts must be performed in real time and that it certainly cannot be performed by using the same sample period for the estimation. We therefore investigate the forecasting ability of model over the 1990–2014 period, using a rolling window of 35 years for the parameter estimation. The first sample covers the period from 1955 to 1989, the last sample covers the period from 1979 to 2014). This period is essentially stationary. To obtain relevant measures in the out-of-sample investigation, we restrict the investment horizon to a maximum of 15 years.

3.3 VAR Model

As indicated before, a difficulty in constructing a macro-finance VAR model is non-stationarity. Some of the variables are nearly non-stationary and may imply explosive dynamics, which would result in unrealistic forecasts of financial returns for long horizons. In particular, the first-order autocorrelation is equal to 0.99 for detrended hours over the full sample. To control for this issue, our benchmark VAR model is estimated with detrended hours in difference. The model is composed of the following variables: real GDP growth (Δy_t), real consumption growth (Δc_t), real investment growth ($\Delta \iota_t$), hours growth (Δh_t), wage inflation (Δwp_t), price inflation (π_t), real federal funds rate ($r_{1,t}$), long-term versus short-term spread ($sp_{b,t} = y_{b,t} - r_{1,t}$), long-term holding-period excess returns ($xr_{b,t} = r_{b,t} - r_{1,t}$), excess stock returns ($xr_{s,t} = r_{s,t} - r_{1,t}$), and log dividend-price ratio ($dpr_t = vp_t - dp_t$). We follow Campbell et al. (2003) in the definition of the financial variables. We define the vector of 11 observable (and state) variables $x_t^{(VAR)} = (\Delta y_t, \Delta c_t, \Delta \iota_t, \Delta h_t, \Delta wp_t, \pi_t, r_{1,t}, sp_{b,t}, xr_{b,t}, xr_{s,t}, dpr_t)'$. The VAR model is therefore written as:

$$x_{t+1}^{(VAR)} = \mu_V + G_V \; x_t^{(VAR)} + \eta_{t+1}^{(VAR)}, \tag{5}$$

where G_V is a matrix of unknown parameters.

The number of parameters in the benchmark VAR is equal to 176 (121 for the autoregressive matrix and 55 for the covariance matrix). The increase in the number of parameters compared to the DSGE model reflects the risk of over-parametrization in the use of VAR models. The parameter estimates of the benchmark VAR model are reported in **Table 2**. We first note that only a small proportion of the parameters are statistically significant. Three variables have relatively large autoregressive terms, the dividend price ratio, the term spread, and the real short-term rate. However, the parameters are close to 0.9, which suggests that the model is stationary. For some macroeconomic variables, such as wages, hours, or inflation, only one or two variables (in addition to the autoregressive term) have some explanatory power, which results in a relatively low adjusted R^2. We also observe that the real excess stock return is mainly driven by the dividend-price ratio (with an adjusted R^2 equal to 11%), whereas the dividend-price ratio itself is mainly driven by its own lag, the term spread, and the inflation rate. Predicting stock returns is therefore likely to be a difficult task in the VAR model. The bond holding-period return is essentially driven by the inflation rate, the short-term rate, and its own lag, with an adjusted R^2 of 23%.

Examining the correlation matrix of residuals also reveals that some residual series are contemporaneously correlated. For the long-term prediction of financial returns in a long-term perspective, an important correlation is that between the residuals of the stock returns and the dividend-price ratio equations. A large negative value of this correlation is cited by Barberis (2000) as a key element for generating mean reversion in the volatility of stock returns as the horizon increases. In our sample, this correlation is equal to −65%. Although this value is large and negative, it is much smaller in absolute value than the correlation found in previous papers over older samples. For instance, Barberis (2000) finds a correlation coefficient equal to −93%. Our estimate suggests that the mean reversion in the volatility of stock returns may be less pronounced over the recent period than reported in previous contributions.

It should be noticed that the estimation method might play a role in explaining differences between the DSGE and VAR models. Because of their complexity, DSGE models are usually estimated using Bayesian techniques. In contrast, most papers investigating the use of VAR models for dynamic asset allocation rely on OLS estimation.[6] The use of different estimation techniques might generate a bias in favor of one of the models. To take this possible bias into account, we also estimate the VAR model with the Bayesian approach. In our specification, all coefficients have zero prior mean except the own lag. As Table 3 shows, in all cases, due to the relatively large sample size, the parameters estimated using the Bayesian technique are close to those obtained with the standard OLS approach.

In an alternative specification, we have replaced hours growth with hours in levels. The parameter estimates of this specification are reported in Table A4. As the table shows, the hours in level exhibit a unit root (with an autoregressive term equal to 0.9975). In Table A5, we also explored a specification with financial variables only, as in Campbell et al. (2003), with $x_t = (r_{1,t}, xr_{s,t}, xr_{b,t}, r_{1,t}^{(n)}, dpr_t, sp_{b,t})'$. In this specification, in our sample, the nominal rate and the dividend price ratio exhibit a unit root (with autoregressive terms equal to 1.018 and 1.006, respectively). We also observe that the R^2 of the excess stock return equation is equal to 4.3% only, which reveals low predictability of the stock return in this model. These results suggest that these alternative specifications are likely to generate long-term predictions of financial returns with explosive patterns.

[Insert Tables 2 and 3 here]

[6] An important exception is Barberis (2000), who precisely use Bayesian techniques to investigate the effect of parameter uncertainty on the optimal allocation. A related paper is Kandel and Stambaugh (1996), who explore the theoretical implications on the optimal allocation when the investor has beliefs against predictability.

3.4 Term Structure of Risks

In our investigation of the properties of the competing models regarding financial return forecasts, we start with the analysis of the term structure of risks. As pointed out by Siegel (1994), Barberis (2000), and Campbell and Viceira (2002), the predictability of stock returns combined with the negative correlation between stock errors and dividend-price ratio errors implies that theannualized volatility of stock returns tends to decreases as the horizon increases. From the point of view of the investment strategy, this result implies that stocks are safer at longer horizons, meaning that a longer-horizon investor should allocate more to stocks. In contrast, the volatility of bond returns tends to increase with the horizon.[7] To investigate the implications of the predictability of financial returns in DSGE and VAR models from an investor perspective, we consider the relative levels of risk of financial assets at various horizons.

We denote by $\tilde{z}_{t+1} = (r_{1,t+1}, r_{b,t+1}, r_{s,t+1})'$ the vector of real log returns. The distribution of the k-period cumulative return, $Z_t[k] = \tilde{z}_{t+1} + \cdots + \tilde{z}_{t+k}$, is given by a $N(\mu_{Z,t}[k], \Sigma_Z[k])$, where

$$
\begin{aligned}
\mu_{Z,t}[k] &= k\tilde{\Phi}_0 + \tilde{\Phi}_1(G + \cdots + G^k)s_t, \\
\Sigma_Z[k] &= \tilde{\Phi}_1 D\tilde{\Phi}_1' + \tilde{\Phi}_1(I+G)D(I+G)'\tilde{\Phi}_1' + \cdots \\
&\quad + \tilde{\Phi}_1(I + G + \cdots + G^{k-1})D(I + G + \cdots + G^{k-1})'\tilde{\Phi}_1',
\end{aligned}
$$

where $D = H\Sigma_\eta H'$. The term $\tilde{\Phi}_0$ contains the long-term values of the real log returns, $\tilde{\Phi}_1$ is the selection matrix that selects the vector of real log returns from the state observable. Finally, we compute the annualized covariance matrix of the k-period cumulative returns as $(1/k)\Sigma_Z[k]$.

Figure 2 plots the annualized volatilities of real returns for horizons up to 120 quarters (i.e., 30 years) as of 2014Q4 for the full DSGE model. Panel A clearly shows that in the full DSGE model, the risk of long-term bonds and stocks decreases with the investment

[7] The decrease in stock return volatility in the long run has been recently questioned by Pástor and Stambaugh (2012), who argue that the negative effect of mean reversion on long-run volatility is more than offset by the uncertainty about future expected returns and parameter uncertainty.

horizon. The annualized volatility of long-term bonds decreases from 8% for a one-year horizon to less than 5.6% for a 30-year horizon. The mean reversion in stock returns is even more pronounced: the volatility of stock returns decreases from 12.5% for a one-year horizon to less than 4% for a 30-year horizon. From an investment perspective, this result implies that long-term bonds and stocks are relatively safer in the long run and that stocks are in fact safer than bonds. Thus, a long-horizon investor should allocate relatively more to stocks and bonds than a short-horizon investor (Barberis, 2000, Campbell et al., 2003). Regarding the short-term bond, we find that the risk continuously increases with the horizon, from 2% to 4%. This pattern reflects the increase in inflation risk for long horizons, due to its weak mean reversion. This result is broadly consistent with the evidence reported by Campbell and Viceira (2002).

Panel B plots the term structure of risks for the restricted DSGE model, where the risk premia are modeled as pure ARMA(1,1) processes. Clearly, the long-term risks differ from those obtained from the full DSGE model. For instance, for the 30-year horizon, the risks are higher for short-term bonds (5.7% vs. 4%), long-term bonds (11% vs. 5.6%), and stocks (6.1% vs. 4%). Although the ranking of risks is preserved, long-term bonds are perceived as considerably riskier for long horizons than in the full DSGE model. This result indicates that a more precise description of the risk premia allows us to reduce these risks for long horizons, even if they are barely altered for short horizons.

[Insert Figure 2 here]

Figure 3 (Panel A) shows the term structure of risks obtained with the VAR model for 2014Q4. We find that in contrast to the DSGE model, the annualized volatility of stocks does not decrease with the forecast horizon in the VAR model, indicating that mean reversion in stock returns is in fact much weaker in a VAR model. The risk of long-term bonds exhibits some mean reversion, but it decreases much more slowly in the VAR model than in the DSGE model. For the 30-year horizon, the annualized volatility estimates in the VAR model are equal to 7.2%, 5.3%, and 15.7% for short-term bonds, long-term bonds, and stocks, respectively (4%, 5.6%, and 4% in the DSGE model).

Interestingly, we find that, in the model with structural economic restrictions, annualized volatilities decrease with the horizon, as suggested by Siegel (1994) and Campbell and Viceira (2002). In contrast, in the unrestricted model, annualized volatilities do not decrease, as established by Pástor and Stambaugh (2012). These results clearly confirm that imposing economic restrictions in a forecasting model might be beneficial from a long-term perspective.

We experienced with several other dates and found essentially the same patterns. In particular, the decrease in the term structure of stock and bond risks in the DSGE model holds true for all of the dates we considered. The increase in the term structure of stock risk and the slow decrease in the term structure of bond risk in the VAR model also hold for all of the dates we considered. In Panel B, we represent the term structure of risks obtained with the Bayesian VAR. We observe the the general patters obtained with the OLS-VAR still hold with the model: the volatility of the stock return does not decrease with the horizon, the volatility of long-term bonds only slowly decreases with the horizon, and the volatility of short-term bonds strongly increases with the horizon.

Panel B illustrates the term structure of risks when the VAR model is estimated using Bayesian techniques with priors against predictability. As it clearly shows, the general patterns are barely changed. For long horizons, the volatility of the stock return is slightly increased whereas the volatility of the long-term bond return is slightly decreased.

We also investigated the term structure of risks implied by the VAR model when we use hours in levels (h_t) instead of growth in hours worked (Δh_t). In fact, changing the specification of the VAR model does not alter the main result. In the VAR formulated by Campbell et al. (2003) with financial variables only, the volatility of the stock return increases to even higher levels (16% for the 30-year horizon). These indicates indicate that the increasing pattern of the volatility of the stock return is not due to the choice of the macro variables. Instead, it seems that the higher correlation between the residuals of the stock return and dividend price ratio equations (−67% instead of −73% in Campbell et al., 2003) is responsible for the non-decreasing term structure of stock risks.

[Insert Figure 3 here]

3.5 Out-of-sample RMSE

A DSGE model can be viewed as a restricted VAR process, which incorporates the restrictions imposed by the macro-finance mechanisms of the model. Thus, an unrestricted VAR may perform better than a DSGE model in the short term because it provides a better fit for the next period. The VAR model could still outperform the DSGE model in the long term if the economic restrictions turn out to be irrelevant. However, if the economic restrictions that are imposed in the DSGE model are relevant, the VAR model may not perform as well as the DSGE model in the long term.

To evaluate the strengths and weaknesses of the DSGE and VAR models in a long-term perspective, we now consider the ability of these models to predict the variables of interest over long horizons. To obtain relevant measures in the out-of-sample investigation, we restrict the forecast horizon to a maximum of 15 years. We estimate the models from 1955Q1 to 1989Q4 and forecast the variables over horizons ranging from one quarter to 15 years. We then roll the sample by one quarter, reestimate the models from 1955Q2 to 1990Q1, and forecast all the variables over the same horizons. We continue this procedure until we reach the last window, 1975Q4 to 2014Q3, for which we forecast for the next quarter only. The number of forecasts is equal to $n = 81$ for the 5-year horizon and to $n = 41$ for the 15-year horizon.

The root mean square error (RMSE) is computed by comparing the k-period cumulative expected real return, $\mu_{Z,t}[k]$, to the ex post observed cumulative real return, $Z_t[k]$. To evaluate the difference between DSGE and VAR forecasts, we use the Diebold and Mariano (1995) test statistics: for a given horizon k, it is defined as $DM_i[k] = \bar{d}_i[k]/\sqrt{\bar{\sigma}_i[k]^2/n}$, where $\bar{d}_i[k]$ is the sample mean and $\bar{\sigma}_i[k]^2$ the sample variance of the loss difference defined as $d_{i,t+k} = (Z_t[k] - \mu_{Z,t}[k]^{(VAR)})^2 - (Z_t[k] - \mu_{Z,t}[k]^{(DSGE)})^2$, with $i = 1, b, s$.[8] Under the null hypothesis that the two forecasts are equal, the test statistic is asymptotically distributed as $N(0, 1)$.

Table 4 reports the RMSE and the Diebold and Mariano (1995) test statistics for the difference between the DSGE and VAR forecasts. We begin with the full DSGE model

[8]We correct the sample variance for overlapping in the loss difference using Newey-West correction.

(Panel A). The RMSE of the real short-term interest rate strongly increases with the horizon, from 8.7% for a 5-year horizon to 19.8% for a 15-year horizon. The large RMSE for long-term horizons can be explained by the use of a Taylor-type rule to describe the central bank's monetary policy. The Taylor rule serves to anchor of the short-term rate to the output gap and the inflation rate. As the inflation rate has weak mean reversion, forecasting the short-term rate is a challenging task. In addition, changes in the short-term rate are often viewed as political decisions. It is also likely that the model has difficulty in generating good forecasts over the recent period because of the low short-term interest rate environment.[9]

The forecast error for the real long-term bond return is large in the short term but decreases for long horizons (from 17.4% for a 5-year horizon to 10.7% for a 15-year horizon). This result suggests that mean reversion contributes substantially to reducing long-term uncertainty. We obtain such a mean reversion only with the full DSGE model, as we observe in subsequent panels. For instance, the RMSE of long-term bond return in the restricted DSGE model is equal to 19.8% for a 15-year horizon, which is nearly twice as large as the RMSE obtained from the full DSGE model. With VAR models, the RMSE of the long-term bond return is even higher and equal to 28.2% for the benchmark model.

Finally, the RMSE is rather large for real stock returns and approximately equal to 32% for horizons between 5 and 15 years. This result is expected given the high uncertainty surrounding stock return forecasts. Even if the model can be considered to provide good overall forecasts, the current version of the model does not account, for instance, for stock market bubbles or for departure from the dividend discount model. This (out-of-sample) finding is consistent with the substantial (in sample) mean reversion reported above and found by Siegel (1994), Barberis (2000), and Campbell and Viceira (2002).

[9]The zero lower bound is clearly an issue for forecasting short-term rates because the level of the output gap and inflation may justify a negative rate and therefore induce a large under-estimation of the actual rate. We investigated the forecast error in the recent period and did not observe any significant increase in the RMSE in this period. A possible explanation is that several subperiods in our sample have been characterized by large over- or under-estimations of the short-term rate, for instance in 1982–1983 and 2000–2001.

The RMSE of the financial returns for the benchmark VAR model, estimated by OLS, are reported in Panel C. The table reveals that the VAR model has performances similar to the DSGE model for the 5-year horizon but underperforms for longer horizons. The main reason for this result is that the VAR model fails at producing any mean reversion in forecasting real bond and stock returns in the long term. For the 10-year horizon, the RMSE for the VAR model increases to 44% (compared to 32% for the DSGE model) for real stock returns. In relative terms, the performance of the DSGE model is even more pronounced for real bond returns. The RMSE of the DSGE model slowly decreases as the horizon increases (from 17.4% for 5 years to 10.7% for 15 years), while the RMSE of the VAR model dramatically increases from 21.4% for 5 years to 28.2% for 15 years. Panel D reports the results for the Bayesian VAR model.

In the specifications of the VAR model with hours in level or with financial variables only, our main result is not altered: although the VAR model performs relatively well in the short run, it clearly underperforms the (full and restricted) DSGE model in forecasting real bond and stock returns in the long run. This result is due to the near-to-unit root dynamics that generate extreme dynamics of financial return forecasts.

The Diebold and Mariano (1995) test statistics for the null hypothesis that the RMSE values are the same for the DSGE and VAR models are also reported. They show that for the 5-year horizon the restricted DSGE and VAR models perform slightly better than the full DSGE model for the long-term bond returns and, to a lesser extent, for the short-term bond returns. In contrast, for 10-year and 15-year horizons, the DSGE model statistically outperforms its competitors for long-term bond and stock returns.

These empirical results are consistent with the idea that imposing economic restrictions improves predictions of financial returns. For instance, Campbell and Thompson (2008), Cochrane (2008), and Ferreira and Santa-Clara (2011) develop a similar argument in a regression context. These results are particularly important from an investment perspective because, as Campbell and Thompson (2008) also show, even a relatively small improvement in prediction can result in meaningful utility gains for investors.

[Insert Table 4 here]

4 Optimal Allocation

We start our analysis of the optimal allocations generated by the DSGE and VAR models with a discussion of the hedging demands implied by these models. Hedging demands allow us to understand how the forecasting models affect the optimal allocation because they involve all the properties of the models through the autoregressive matrix G and the covariance matrix Σ_η. Then, we evaluate the financial performance of the forecasting models in an out-of-sample perspective.

4.1 Hedging Demands

As a first step of our analysis of the optimal strategy of the institutional investor, we consider the total demands for stocks and bonds and their myopic and hedging decomposition. **Figure** 4 displays the optimal demands based on the full DSGE model and the VAR model when the state variables are taken at their sample average. For the allocation based on the DSGE model (Panel A), the figure shows that the allocation to stocks is a concave function of risk tolerance ($1/\gamma$), as in Campbell et al. (2003), with a similar decomposition in its myopic and hedging components. The intertemporal hedging demand is always positive and is the highest for intermediate levels of risk tolerance: it is above 50% for risk aversion levels between 2 and 10, and then decreases to nearly 0 for infinitely risk-averse investors. Regarding the allocation to bonds, we also notice a concave function of the risk tolerance, although the hedging demand is more limited. The highest hedging demand is equal to 10% for a risk aversion of $\gamma = 5$. For an infinitely risk-averse investor, the optimal portfolio is composed of 15% of equity and 85% of cash.

The optimal allocation based on the VAR model exhibitsvery different patterns (Panel B). The hedging demand for stocks is always negative with a minimum of -160% for intermediate levels of risk aversion. As the myopic allocation is positive and decreasing, we observe that the total demand for stocks is positive for low level of risk aversion (up to 2), and becomes negative for intermediate and high levels of risk aversion, with a minimum of -60% for $\gamma = 5$. Finally, for infinitely risk-avers investors, the optimal weights of stocks is 0. The myopic demand for bonds is positive and decreasing as for stock. However, the hedging demand increases for intermediate levels of risk aversion, with a maximum of 300% for $\gamma = 4$. For infinitely risk-avers investors, the optimal portfolio is composed of 140% of bonds and -40% of cash.

[Insert Figure 4 here]

To investigate in greater detail the role of the risk premia in the optimal allocation based on the DSGE model, we consider the deviations from the current stock premium (by -5% and +5%) when the bond premium is at its average value. As **Figure 5** (Panel A) shows, as the stock premium increases, the allocation to stocks decreases. The highest hedging demand is 80% when the stock premium is -5% and 50% when the premium increases to 5% (in both cases, when γ is close to 5). This result is consistent with the negative correlation of stock returns with the stock premium and indicates that stocks are good hedges against stock risk.[10] This point was already made by Campbell et al. (2003) in the context of a VAR model: stock return at t is negatively correlated with the dividend-price ratio at t, whereas a high dividend-price ratio at t predicts a higher stock return at $t+1$. Therefore, a low stock return today is associated to a higher stock return tomorrow. As a consequence, a low stock premium today commands an increase in the optimal weight of stocks. However, this relation does no longer hold in the VAR model over our sample because the correlation between current stock return and dividend-price ratio is much less negative in the recent period. We also notice that the hedging demand for bonds is barely affected by the stock premium.

Panel B corresponds to deviations from the current bond premium (by -5% and +5%) when the stock premium is at its average value. For a negative premium, we find that the hedging demand for bonds is positive and large (up to 25%) for low levels of risk aversion. Similar results are reported by Sangvinatsos and Wachter (2005) for long-term investors. As bond returns are positively correlated with the bond premium,

[10] In our sample, the correlation of the stock return is equal to -0.04 with the stock risk premium and -0.19 with the bond risk premium. The correlation of the bond return is equal to -0.08 and 0.26 with the stock and bond risk premia, respectively. The correlation of the cash return is equal to -0.2 and 0.43 with the stock and bond risk premia, respectively.

a negative bond premium induces a positive hedging demand for bonds. However, long-term investors know that the bond premium is positive, on average, and thus expect the risk premium to return to its long-run value in the future. Therefore, risk-averse investors still hold long positions in bonds. As the bond premium increases, the allocation to bonds decreases accordingly. For a premium of 5%, the hedging demand decreases to 15% for intermediate risk aversion parameters. Regarding the allocation to stocks, we find that, as the bond premium increases, the fraction of wealth invested to stocks is not significantly affected.

[Insert Figure 5 here]

4.2 Out-of-sample Optimal Allocations

We now evaluate the out-of-sample performance of the DSGE and VAR models from a long-term investment perspective. We adopt the same estimation strategy as for computing the RMSE. Over the full sample, we use the first 35 years (1955–1989) for the first estimation of the (DSGE and VAR) models and the sample for the 1990–2014 period for the performance evaluation. We proceed as follows: for the cohort of investors who begin to invest in 1990Q1, we estimate the model over the 1955–1989 period and determine the optimal rule (Equation (1)) by using available data only. Then, every quarter from 1990Q1 to 2014Q3, we compute the optimal weights of their portfolios conditional on the available state variables. Finally, once asset returns are observed, we compute the ex-post portfolio return, from which we evaluate the out-of-sample performance of the investment rule. For the 1990Q1 cohort, we have 98 optimal portfolio weights and therefore 98 ex-post portfolio returns. We then move to the next cohort of investors, who begin to invest in 1990Q2, and proceed similarly. For the last cohort (2014Q3), only one optimal allocation and one ex-post portfolio return are available. As the various cohorts use different estimated models, the investment rules and, consequently, the ex-post performances differ from one cohort to the other.

In **Tables 5** and **6**, we report the average portfolio weights for stocks and bonds for the DSGE and VAR models. For instance, for the 5-year horizon, we average the optimal portfolio weights of the cohorts from 1990Q1 to 2009Q3. In doing so, we avoid overreliance on a specific period, as we average the behavior of different investors' cohorts over time. Over longs horizons, in the optimal DSGE portfolio, the demand is on average positive for both stocks and bonds. In contrast, in the optimal VAR portfolio, the demand is positive for stocks but large and negative for bonds. This clear difference between the optimal DSGE and VAR portfolios can be explained as follows. As noted above, the DSGE model provides much more accurate forecasts of long-term bond returns than the VAR model; thus, the investor's perceived risk on long-term bonds is much lower. Although the DSGE model is also better at reducing the uncertainty surrounding stock returns than the VAR model, the decrease in risk is relatively less pronounced. As a consequence, the myopic demand is positive for bonds and stocks for the DSGE investor and positive for stocks and negative for bonds for the VAR investor. In addition, as discussed in the previous section, in the DSGE model, the hedging demand is positive for stocks, owing to their ability to hedge changes in the stock risk premium, and relatively limited for bonds. In fact, the demand for bonds is almost entirely financed by a negative demand for cash, which has a large positive correlation with the bond premium and does help with hedging this source of risk.

The difference in portfolio weights between the DSGE and VAR investors can be visualized in **Figures 6** and **7**, which show the average portfolio weights for a given date across all the cohorts that invest on that date (with $\gamma = 5$ and 20, respectively). For instance, the weights for 2000Q1 correspond to the average of the weights for all the cohorts from 1990Q1 to 2000Q1 (assuming equal weights across cohorts). As Panel A clearly illustrates, the DSGE investor invests in stocks with a countercyclical dynamics: the demand for stocks decreases before the Internet bubble burst and before the subprime crisis. These episodes correspond to periods with a large decrease in the stock premium. The figure also indicates that the hedging demand for bonds is negative most of the time. This result reflects that, with the DSGE model, investing in bonds is a bad hedge against changes in the bond risk premium due to the positive correlation of bond returns with the bond risk premium.

As Panel B shows, the hedging demand for bonds is even lower for the VAR investor. The VAR model generates hedging (and total) demands for bonds that are negative, while the investor is usually long in stocks. Contrary to the DSGE investor, the VAR investor is not countercyclical in the allocation to stocks. In particular, she does not reduce the allocation to stocks before the subprime crisis. In addition, the demand for cash is very large.

The out-of-sample performances of the dynamic strategies are also reported in Tables 5 and 6. As can be seen in Panel C, the annualized (real) return of the DSGE portfolio is positive for all horizons and all levels of risk aversion. For the 10-year horizon, the average real return is approximately 19% per year for low risk aversion ($\gamma = 5$), and this value decreases to 8.2% for $\gamma = 10$ and 4% for $\gamma = 20$. The annualized volatility is also relatively high, particularly for low risk aversion and long horizons (Panel D). The Sharpe ratio ranges between 0.2 and 0.3, a relatively high range of values given the limited investment set (Panel E).[11] Sharpe ratios are similar although lower, for the restricted DSGE model.

Turning to the VAR portfolio, we notice that the annualized return is relatively low for all levels of risk aversion and all horizons. This poor performance primarily occurs because the VAR portfolio is long in stocks at the beginning of the subprime crisis and therefore generates large losses, whereas the DSGE portfolio has zero exposure to the stock market. A similar, although less penalizing, situation occurred at the eve of the Internet bubble burst, when the DSGE portfolio is short in stocks, whereas the VAR portfolio has zero exposure to the stock market. As the annualized volatility is also relatively high for the VAR portfolio, similar to the DSGE portfolio volatility, the Sharpe

[11] As returns are measured quarterly and the investment horizons are from 5 to 15 years, the Sharpe ratio must be computed carefully. We follow Lo (2002) approach and recognize that returns are not i.i.d. The Sharpe ratio for a k-year horizon is therefore defined as: $SR[k] = SR[1]\eta[k]$, where $SR[1] = (\bar{r}_p - \bar{r}_1)/\sigma_p$ denotes the 1-quarter Sharpe ratio and $\eta[k] = k/(k + 2\sum_{i=1}^{k-1}(k-i)\varphi_i)^{1/2}$ is the scale factor that corrects for the fact that returns are not i.i.d. The approach involves $\varphi_i = Cov[r_{p,t}, r_{p,t-i}]/\sigma_p^2$ the i-th order serial correlation of real portfolio return, and $\sigma_p^2 = V[r_{p,t}]$.

ratio of the VAR portfolio is very low for low and medium levels of risk aversion. The use of OLS or Bayesian estimation techniques does not alter these patterns.

The main explanation for the performance difference between the DSGE and VAR models is that the former is able to provide more accurate forecasts of long-term returns whereas the latter is not. The optimal allocations of the VAR model are broadly consistent with those reported by Campbell et al. (2003), who find portfolios that are long in stocks and short in bonds. In contrast, DSGE portfolios are more in line with the results obtained by Sangvinatsos and Wachter (2005), who allow for a better description of the bond premium. A similar result is also reported by Koijen et al. (2010) but with much more limited hedging demands, owing to short sales restrictions.[12] As Figure 8 indicates, the technique used for the estimation of the VAR models barely affects the dynamics of the portfolio weights.

[Insert Tables 5 and 6 and Figures 6 to 8 here]

5 Conclusion

In this paper, we investigate the ability of a fully structural DSGE model to provide forecasts of future asset returns over long horizons. The model describes the demand and production sides of the economy to obtain the dynamics of the short-term interest rate (set by the central bank), the long-term Treasury bond return, and the stock market return. We also introduce rebalancing costs for risky financial assets, which allow us to generate time-varying risk premia for bonds and stocks. The model shows good performance in forecasting bond and stock returns over long horizons.

The long-term properties of the model are evaluated along two lines. First, we find that the DSGE model is able to generate sufficient mean reversion for bond and stock

[12] Another possible explanation for the overperformance of the DSGE allocation relative to the VAR allocation is that the DSGE model likely mitigates estimation error, which is a well-known major issue in portfolio management. In our context, the unrestricted VAR model involves the estimation of 187 parameters (autoregressive terms and the covariance matrix), whereas the DSGE model involves 67 parameters only. Empirical evidence clearly suggests that imposing theoretical restrictions is beneficial (mitigation of estimation error) rather than detrimental (loss of information).

returns, such that the term structure of risks is decreasing for both asset classes. The VAR models that we consider are not able to produce such long-run mean reversion. Second, we find that the DSGE model provides more accurate long-term out-of-sample forecasts of financial returns than its VAR competitors. The difference in RMSE is large and significant for both bond and stock returns for 10-year and 15-year horizons.

From a long-term allocation perspective, we find that the optimal portfolio should be invested both in stocks and bonds. For high risk aversion levels, however, the hedging demand for stocks is higher than the hedging demand for bonds, resulting in portfolios that are mostly invested in stocks. In our out-of-sample allocation exercise, we find that risk-averse investors will indeed hold stocks and bonds. In contrast, when an unrestricted VAR model is used to forecast returns, the investor will optimally hold stocks and short bonds.

All these findings suggest that the use of a structural macro-finance model, by imposing long-run restrictions on financial returns, provides superior long-term forecasts of financial returns. Therefore, it may be of value for long-term investors, such as insurance companies, pension funds, and sovereign wealth funds.

References

Abel, A., Blanchard, O., 1983. An intertemporal model of saving and investment. Econometrica 51, 675–692.

Alpanda, S., 2013. Identifying the role of risk shocks in the business cycle using stock price data. Economic Inquiry 51, 304–335.

Andrés, J., López-Salido, J., Nelson, E., 2004. Tobin's imperfect asset substitution in optimizing general equilibrium. Journal of Money, Credit, and Banking 36, 665–690.

Ang, A., Piazzesi, M., 2003. A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables. Journal of Monetary Economics 50, 745–787.

Barberis, N., 2000. Investing for the long-run when returns are predictable. Journal of Finance 55, 225–264.

Bekaert, G., Cho, S., Moreno, A., 2010. New-Keynesian macroeconomics and the term structure. Journal of Money, Credit, and Banking 42, 33–62.

Bodie, Z., Detemple, J., Rindisbacher, M., 2009. Life-cycle finance and the design of pension plans. Annual Review of Financial Economics 1, 249–286.

Boldrin, M., Christiano, L., Fisher, J., 2001. Habit persistence, asset returns and the business cycle. American Economic Review 91, 149–166.

Brennan, M., Schwartz, E., Lagnado, R., 1997. Strategic asset allocation. Journal of Economic Dynamics and Control 21, 1377–1403.

Buss, A., Dumas, B., 2012. The equilibrium dynamics of liquidity and illiquid asset prices. INSEAD Working Paper.

Calvo, G., 1983. Staggered prices in a utility-maximizing framework. Journal of Monetary Economics 12, 383–398.

Campbell, J., Chan, Y., Viceira, L., 2003. A multivariate model of strategic asset allocation. Journal of Financial Economics 67, 41–80.

Campbell, J., Thompson, S., 2008. Predicting the equity premium out of sample: Can anything beat the historical average? Review of Financial Studies 21, 1509–1531.

Campbell, J., Viceira, L., 1999. Consumption and portfolio decisions when expected returns are time varying. Quarterly Journal of Economics 114, 433–495.

Campbell, J., Viceira, L., 2001. Who should buy long-term bonds? American Economic Review 91, 99–127.

Campbell, J., Viceira, L., 2002. Strategic asset allocation. Oxford University Press.

Christiano, L., Eichenbaum, M., Evans, C., 2005. Nominal rigidities and the dynamic effects of a shock to monetary policy. Journal of Political Economy 113, 1–45.

Chugh, S., 2006. Optimal fiscal and monetary policy with sticky wages and sticky prices. Review of Economic Dynamics 9, 683–714.

Cochrane, J., 1991. Production-based asset pricing and the link between stock returns and economic fluctuations. Journal of Finance 46, 209–237.

Cochrane, J., 2008. The dog that did not bark: A defense of return predictability. Review of Financial Studies 21, 1533–1575.

Cooper, I., Priestley, R., 2008. Time-varying risk premiums and the output gap. Review of Financial Studies 22, 2802–2833.

De Graeve, F., Dossche, M., Emiris, M., Sneessens, H., Wouters, R., 2010. Risk premiums and macro economic dynamics in a heterogeneous agent model. Journal of Economic Dynamics and Control 34, 1680–1699.

Diebold, F., Mariano, R., 1995. Comparing predictive accuracy. Journal of Business and Economics Statistics 13, 253–264.

Dixit, A., Stiglitz, J., 1977. Monopolistic competition and optimum product diversity. American Economic Review 67, 297–308.

Epstein, L., Zin, S., 1989. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework. Econometrica 57, 937–969.

Falagiarda, M., Massimiliano, M., 2012. A DSGE model with endogenous term structure. Quaderni DSE Working Paper No. 830.

Fernandez-Villaverde, J., Rubio-Ramirez, J., 2004. Comparing dynamic equilibrium models to data: A Bayesian approach. Journal of Econometrics 123, 153–187.

Ferreira, M., Santa-Clara, P., 2011. Forecasting stock market returns: The sum of the parts is more than the whole. Journal of Financial Economics 100, 514–537.

Fuhrer, J., 2000. Habit formation in consumption and its implications for monetary-policy models. American Economic Review 90, 367–390.

Guibaud, S., Nosbusch, Y., Vayanos, D., 2008. Preferred habitat and the optimal maturity structure of government debt. Working Paper. Available at SSRN: http://ssrn.com/abstract=1098195.

Guvenen, F., 2009. A parsimonious macroeconomic model for asset pricing. Econometrica 77, 1711–1750.

Hördahl, P., Tristani, O., Vestin, D., 2008. The yield curve and macroeconomic dynamics. Economic Journal 118, 1937–1970.

Jermann, U., 1998. Asset pricing in production economies. Journal of Monetary Economics 41, 257–275.

Jondeau, E., Sahuc, J.-G., 2008. Optimal monetary policy in an estimated DSGE model of the Euro Area with cross-country heterogeneity. International Journal of Central Banking 4, 23–72.

Kandel, S., Stambaugh, R., 1996. On the predictability of stock returns: An asset-allocation perspective. Journal of Finance 51, 385–424.

Kim, S., Omberg, E., 1996. Dynamic nonmyopic portfolio behavior. Review of Financial Studies 9, 141–161.

Koijen, R., Nijman, T., Werker, B., 2010. When can life cycle investors benefit from time-varying bond risk premia. Review of Financial Studies 23, 741–780.

Lettau, M., Ludvigson, S., 2001. Consumption, aggregate wealth, and expected stock returns. Journal of Finance 56, 815–849.

Lo, A., 2002. The statistics of Sharpe ratios. Financial Analysts Journal 58, 36–52.

Lucas, R., 1976. Econometric policy evaluation: A critique. Carnegie-Rochester Conference Series on Public Policy 1, 19–46.

Lynch, A., 2001. Portfolio choice and equity characteristics: Characterizing the hedging demands induced by return predictability. Journal of Financial Economics 62, 67–130.

Marzo, M., Söderström, U., Zagaglia, P., 2008. The term structure of interest rates and the monetary transmission mechanism. Manuscript, Sveriges Riksbank.

McGrattan, E., Prescott, E., 2005. Taxes, regulations, and the value of U.S. and U.K. corporations. Review of Economic Studies 72, 767–796.

Merton, R., 1969. Lifetime portfolio selection under uncertainty: The continuous-time case. Review of Economics and Statistics 51, 247–257.

Merton, R., 1971. Optimum consumption and portfolio rules in a continuous time model. Journal of Economic Theory 3, 373–413.

Merton, R., 1973. An intertemporal capital asset pricing model. Econometrica 41, 867–887.

Modigliani, F., Sutch, R., 1966. Innovations in interest rate policy. American Economic Review 56, 178–197.

Modigliani, F., Sutch, R., 1967. Debt management and the term structure of interest rates: An empirical analysis of recent experience. Journal of Political Economy 75, 569–589.

Pástor, L., Stambaugh, R., 2012. Are stocks really less volatile in the long run? Journal of Finance 67, 431–478.

Rotemberg, J., 1983. Monopolistic price adjustment and aggregate output. Review of Economic Studies 49, 517–531.

Rudebusch, G., Sack, B., Swanson, E., 2007. Macroeconomic implications of changes in the term premium. Federal Reserve Bank of St. Louis Review 89, 241–269.

Rudebusch, G., Swanson, E., 2008. Examining the bond premium puzzle with a DSGE model. Journal of Monetary Economics 55 (Supplement 1), S111–S126.

Rudebusch, G., Swanson, E., 2012. The bond premium in a DSGE model with long-run real and nominal risks. American Economic Journal: Macroeconomics 4, 105–143.

Samuelson, P., 1969. Lifetime portfolio selection by dynamic stochastic programming. Review of Economics and Statistics 51, 238–246.

Sangvinatsos, A., Wachter, J., 2005. Does the failure of the expectations hypothesis matter for long-term investors? Journal of Finance 60, 179–230.

Santos, T., Veronesi, P., 2006. Labor income and predictable stock returns. Review of Financial Studies 19, 1–44.

Schorfheide, F., 2003. Labor-supply shifts and economic fluctuations. Journal of Monetary Economics 50, 1751–1768.

Siegel, J., 1994. Stocks for the long run. McGraw-Hill, New York.

Smets, F., Wouters, R., 2003. An estimated dynamic stochastic general equilibrium model of the Euro Area. Journal of the European Economic Association 1, 1123–1175.

Smets, F., Wouters, R., 2007. Shocks and frictions in US business cycles: a Bayesian DSGE approach. American Economic Review 97, 586–606.

Van Binsbergen, J., Fernandez-Villaverde, J., Koijen, R., Rubio-Ramirez, J., 2012. The term structure of interest rates in a DSGE model with recursive preferences. Journal of Monetary Economics 59, 634–648.

Vayanos, D., Vila, J.-L., 2009. A preferred-habitat model of the term structure of interest rates. NBER Working Paper 15487.

Wachter, J., 2002. Portfolio and consumption decisions under mean-reverting returns: an exact solution for complete markets. Journal of Financial and Quantitative Analysis 37, 69–91.

Wachter, J., 2010. Asset allocation. Annual Review of Financial Economics 2, 175–206.

Wu, T., 2006. Macro factors and the affine term structure of interest rates. Journal of Money, Credit, and Banking 38, 1847–1875.

Table 1: Summary statistics on observable variables

Variable		Annual. mean (×100)	Annual. std dev. (×100)	Persistence
Panel A: Macro variables				
Change in real GDP	Δy_t	1.68	3.57	0.317
Change in real consumption	Δc_t	1.88	2.78	0.307
Change in real investment	Δx_t	2.39	16.73	0.204
Hours	h_t	–	4.92	0.990
Change in real wages	Δwp_t	1.33	3.17	-0.087
GDP inflation	π_t	3.28	2.28	0.865
Change in real market value of equity	Δvp_t	2.98	30.01	0.047
Change in real dividends	Δdp_t	1.76	31.37	0.129
Dividend-price ratio	dpr_t	–	0.38	0.917
Panel B: Financial variables				
Nominal short-term rate	$r_{1,t}^{(n)}$	5.13	3.53	0.968
Nominal long-term rate	$y_{b,t}^{(n)}$	6.16	2.76	0.986
Nominal bond return	$r_{b,t}^{(n)}$	6.04	12.90	0.274
Nominal stock return	$r_{s,t}^{(n)}$	11.81	29.69	0.050
Real short-term rate	$r_{1,t}$	1.86	2.58	0.850
Real bond return	$r_{b,t}$	2.76	13.26	0.303
Real stock return	$r_{s,t}$	8.53	29.94	0.055
Term spread	$y_{b,t} - r_{1,t}$	1.03	1.58	0.886
Excess bond return	$r_{b,t} - r_{1,t}$	0.90	12.72	0.280
Excess stock return	$r_{s,t} - r_{1,t}$	6.67	30.05	0.068

Note: The table reports the annualized mean, annualized standard deviation, and persistence parameter (first-order autocorrelation) of the observable variables. The data is quarterly from 1955Q1 to 2014Q4, for a total of 240 observations.

Table 2: Parameter estimates of the standard VAR(1) model

	Δy_t	Δc_t	$\Delta \iota_t$	Δh_t	Δwp_t	π_t	$r_{1,t}$	$sp_{b,t}$	$x_{b,t}$	$x_{s,t}$	dpr_t	R^2
Panel A: Parameter estimates												
Δy_{t+1}	-0.31	0.48	0.04	0.22	0.07	0.03	0.04	0.49	-0.01	0.02	0.00	0.31
	(1.98)	(3.67)	(1.33)	(2.85)	(1.06)	(0.24)	(0.45)	(2.70)	(0.32)	(2.92)	(0.33)	
Δc_{t+1}	-0.03	0.12	0.04	0.04	0.11	0.06	0.11	0.43	0.04	0.02	0.00	0.28
	(0.25)	(1.16)	(1.84)	(0.59)	(2.18)	(0.67)	(1.52)	(3.00)	(2.83)	(3.30)	(0.66)	
$\Delta \iota_{t+1}$	-2.15	3.12	0.13	1.49	-0.12	0.34	-0.16	2.12	-0.14	0.09	-0.01	0.40
	(3.12)	(5.46)	(1.09)	(4.41)	(0.43)	(0.70)	(0.40)	(2.68)	(1.89)	(3.19)	(1.30)	
Δh_{t+1}	-0.09	0.51	0.05	0.09	-0.03	0.23	-0.02	0.46	-0.01	0.03	0.00	0.37
	(0.53)	(3.80)	(1.79)	(1.15)	(0.49)	(2.00)	(0.22)	(2.48)	(0.64)	(4.06)	(0.00)	
Δwp_{t+1}	0.12	0.12	-0.01	-0.06	-0.14	-0.15	-0.10	-0.30	0.03	0.02	0.00	0.09
	(0.72)	(0.87)	(0.33)	(0.79)	(2.12)	(1.30)	(1.03)	(1.61)	(1.70)	(2.36)	(1.08)	
π_{t+1}	0.07	-0.02	-0.02	0.04	0.02	0.83	0.05	-0.10	0.00	0.00	0.00	0.77
	(1.13)	(0.39)	(1.82)	(1.53)	(0.66)	(19.9)	(1.60)	(1.55)	(0.54)	(0.87)	(0.21)	
$r_{1,t+1}$	-0.02	0.06	0.01	-0.01	-0.02	0.21	0.87	0.14	-0.01	0.00	0.00	0.75
	(0.24)	(1.04)	(0.90)	(0.19)	(0.59)	(4.34)	(22.4)	(1.81)	(1.57)	(0.01)	(0.29)	
$sp_{b,t+1}$	-0.02	-0.03	0.00	-0.03	0.00	-0.03	0.02	0.89	0.01	0.00	0.00	0.82
	(0.46)	(1.08)	(0.33)	(1.98)	(0.10)	(1.02)	(1.13)	(22.0)	(2.17)	(0.06)	(0.30)	
$x_{b,t+1}$	-0.93	-0.32	0.16	-0.03	0.05	-0.47	1.24	2.71	0.20	-0.07	0.00	0.23
	(1.52)	(0.63)	(1.53)	(0.10)	(0.21)	(1.09)	(3.52)	(3.89)	(3.01)	(2.73)	(0.50)	
$x_{s,t+1}$	0.77	-0.77	-0.12	-0.30	0.11	-1.07	-0.75	1.27	0.33	0.03	0.06	0.11
	(0.51)	(0.61)	(0.45)	(0.40)	(0.18)	(0.99)	(0.85)	(0.72)	(2.02)	(0.49)	(3.63)	
dpr_{t+1}	-0.19	0.23	-0.09	1.04	-0.53	3.36	-0.60	6.44	-0.43	0.08	0.92	0.89
	(0.10)	(0.13)	(0.24)	(1.05)	(0.64)	(2.33)	(0.51)	(2.76)	(1.97)	(0.96)	(39.9)	
Panel B: Correlation matrix of residuals												
Δy_t	1	0.64	0.74	0.53	0.03	-0.18	0.27	-0.07	-0.25	0.09	0.26	
Δc_t	–	1	0.17	0.34	0.18	-0.17	0.24	-0.08	-0.16	0.12	0.06	
$\Delta \iota_t$	–	–	1	0.53	-0.05	0.01	0.09	-0.03	-0.24	0.00	0.31	
Δh_t	–	–	–	1	-0.04	0.11	0.07	-0.11	-0.31	0.04	0.16	
Δwp_t	–	–	–	–	1	-0.13	0.08	0.05	0.03	0.02	-0.07	
π_t	–	–	–	–	–	1	-0.80	-0.07	-0.11	-0.10	0.08	
$r_{1,t}$	–	–	–	–	–	–	1	-0.45	-0.22	0.01	-0.05	
$sp_{b,t}$	–	–	–	–	–	–	–	1	-0.03	0.14	0.01	
$x_{b,t}$	–	–	–	–	–	–	–	–	1	0.00	-0.05	
$x_{s,t}$	–	–	–	–	–	–	–	–	–	1	-0.65	
dpr_t	–	–	–	–	–	–	–	–	–	–	1	

Note: The table reports the parameter estimates (Panel A) and the correlation matrix of residuals (Panel B) for the unrestricted VAR(1) model. Numbers in parentheses represent the t-stat of the parameter estimates. We use the notations $sp_{b,t} = y_{b,t} - r_{1,t}$, $x_{b,t} = r_{b,t} - r_{1,t}$, $x_{s,t} = r_{s,t} - r_{1,t}$, and $dpr_t = dp_t - vp_t$.

Table 3: Parameter estimates of the Bayesian VAR(1) model

	Δy_t	Δc_t	$\Delta \iota_t$	Δh_t	Δwp_t	π_t	$r_{1,t}$	$sp_{b,t}$	$x_{b,t}$	$x_{s,t}$	dpr_t
Δy_{t+1}	-0.36	0.62	0.04	0.29	0.05	0.01	0.09	0.29	0.01	0.01	0.00
	(0.20)	(0.16)	(0.89)	(0.21)	(0.21)	(0.07)	(0.09)	(0.04)	(0.79)	(2.03)	(2.67)
Δc_{t+1}	-0.22	0.26	0.06	0.11	0.07	0.08	0.24	0.48	0.06	0.01	0.00
	(0.17)	(0.14)	(0.75)	(0.17)	(0.18)	(0.06)	(0.07)	(0.04)	(0.66)	(1.65)	(2.21)
$\Delta \iota_{t+1}$	-1.61	3.56	0.13	1.52	0.04	0.58	-0.66	0.57	-0.08	0.07	0.00
	(0.04)	(0.03)	(0.16)	(0.04)	(0.04)	(0.01)	(0.02)	(0.01)	(0.13)	(0.35)	(0.46)
Δh_{t+1}	0.17	0.48	-0.01	0.21	-0.02	0.22	-0.07	0.27	0.01	0.03	0.00
	(0.11)	(0.08)	(0.45)	(0.11)	(0.11)	(0.04)	(0.04)	(0.02)	(0.40)	(0.99)	(1.35)
Δwp_{t+1}	0.29	-0.06	-0.01	-0.19	-0.17	-0.19	0.15	-0.09	0.03	0.01	0.00
	(0.09)	(0.07)	(0.39)	(0.09)	(0.09)	(0.03)	(0.04)	(0.02)	(0.34)	(0.86)	(1.11)
π_{t+1}	0.10	-0.04	-0.02	0.04	0.02	0.84	0.04	-0.01	0.00	0.00	0.00
	(0.15)	(0.12)	(0.66)	(0.16)	(0.15)	(0.1)	(0.06)	(0.04)	(0.59)	(1.43)	(1.92)
$r_{1,t+1}$	-0.09	0.16	0.01	0.02	-0.02	0.29	0.85	0.07	-0.01	0.00	0.00
	(0.13)	(0.10)	(0.54)	(0.13)	(0.13)	(0.04)	(0.1)	(0.03)	(0.49)	(1.19)	(1.60)
$sp_{b,t+1}$	0.03	-0.09	0.00	-0.05	-0.01	-0.10	0.05	0.87	0.00	0.00	0.00
	(0.25)	(0.19)	(1.07)	(0.25)	(0.25)	(0.09)	(0.11)	(0.1)	(0.98)	(2.39)	(3.21)
$x_{b,t+1}$	-0.74	-0.91	0.19	0.10	0.07	-0.92	1.52	2.56	0.19	-0.10	0.00
	(0.02)	(0.02)	(0.10)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.09)	(0.22)	(0.29)
$x_{s,t+1}$	1.31	0.36	-0.23	0.09	0.50	-2.07	-1.89	-3.02	0.20	0.05	0.08
	(0.01)	(0.01)	(0.04)	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)	(0.03)	(0.09)	(0.12)
dpr_{t+1}	-1.27	-0.70	0.08	1.16	-1.36	6.94	0.88	10.30	-0.27	0.06	0.88
	(0.00)	(0.00)	(0.01)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.01)	(0.02)	(0.0)

Note: The table reports the parameter estimates for the VAR(1) model estimated with Bayesian technique. Numbers in parentheses represent the interquartile range of the parameter estimates. We use the notations $sp_{b,t} = y_{b,t} - r_{1,t}$, $x_{b,t} = r_{b,t} - r_{1,t}$, $x_{s,t} = r_{s,t} - r_{1,t}$, and $dpr_t = dp_t - vp_t$.

Table 4: RMSE of DSGE and VAR models

	RMSE			Diebold-Mariano statistics		
	5 years	10 years	15 years	5 years	10 years	15 years
Panel A: Full DSGE model						
Short-term rate	8.75	14.46	19.84			
Bond return	17.40	13.43	10.75			
Stock return	31.77	32.26	32.28			
Panel B: Restricted DSGE model						
Short-term rate	8.40	14.92	22.99	-0.98	0.56	1.75
Bond return	15.42	15.64	19.76	-1.78	0.96	3.71
Stock return	34.55	35.60	33.03	1.10	1.68	0.99
Panel C: Standard VAR model						
Short-term rate	7.23	13.22	20.64	-2.81	-1.98	1.11
Bond return	21.44	25.38	28.19	1.31	7.43	12.52
Stock return	35.74	44.07	31.89	0.97	3.24	-0.39
Panel D: Bayesian VAR model						
Short-term rate	7.23	13.57	20.73	-3.09	-2.06	0.54
Bond return	22.14	25.70	28.65	1.43	6.03	13.52
Stock return	34.66	40.85	31.09	0.54	2.52	-1.58
Nb of observations	81	61	41			

Note: The table reports the percent RMSE (root mean square error) and the Diebold and Mariano (1995) test statistics for the difference between the DSGE and VAR RMSE, over the 5, 10, and 15-year horizons in the DSGE model and VAR(1) models. The Diebold and Mariano (1995) test statistics for the difference between two forecasts at horizon k is: $DM_i[k] = \bar{d}_i[k]/\sqrt{\bar{\sigma}_i[k]^2/n}$, where $\bar{d}_i[k]$ is the sample mean and $\bar{\sigma}_i[k]^2$ the sample variance of the loss difference defined as $d_{i,t+k} = (Z_t[k] - \mu_{Z,t}[k]^{(VAR)})^2 - (Z_t[k] - \mu_{Z,t}[k]^{(DSGE)})^2$, with $i = 1, b, s$.

Table 5: Out-of-sample performance of dynamic strategies – DSGE models

Risk aversion (γ)	Full DSGE model Investment horizon			Restricted DSGE model Investment horizon		
	5 years	10 years	15 years	5 years	10 years	15 years
Panel A: Average demand for stocks						
5	1.29	1.12	1.00	0.97	1.18	1.36
10	0.82	0.76	0.75	0.63	0.75	0.89
20	0.52	0.50	0.52	0.40	0.47	0.57
Panel B: Average demand for bonds						
5	-0.08	0.83	1.80	1.76	3.46	5.37
10	-0.09	0.34	0.80	0.86	1.67	2.56
20	-0.01	0.19	0.41	0.58	0.97	1.39
Panel C: Annualized real return						
5	15.95	18.89	20.87	18.87	23.30	28.19
10	7.03	8.25	8.98	5.86	7.71	9.86
20	3.48	4.02	4.29	1.89	2.75	3.70
Panel D: Annualized volatility						
5	78.90	86.37	94.05	123.78	140.43	167.50
10	42.10	46.40	50.60	66.99	75.31	88.61
20	22.20	24.71	27.11	36.24	40.29	47.08
Panel E: Sharpe ratio						
5	0.24	0.30	0.25	0.17	0.22	0.23
10	0.20	0.28	0.21	0.10	0.14	0.14
20	0.21	0.24	0.21	0.06	0.10	0.11

Note: The table reports the average optimal weight for stocks and bonds and statistics on ex-post performances of the dynamic investment strategies based on the baseline and restricted DSGE models. Return and volatility are in annualized percent.

Table 6: Out-of-sample performance of dynamic strategies – VAR models

Risk aversion (γ)	Standard VAR(1) model Investment horizon			Bayesian VAR(1) model Investment horizon		
	5 years	10 years	15 years	5 years	10 years	15 years
Panel A: Average demand for stocks						
5	1.09	1.12	1.04	1.00	1.04	0.90
10	0.55	0.60	0.57	0.49	0.54	0.47
20	0.25	0.28	0.28	0.21	0.25	0.22
Panel B: Average demand for bonds						
5	-5.25	-6.57	-6.41	-4.69	-6.05	-5.66
10	-2.75	-3.66	-3.75	-2.37	-3.30	-3.21
20	-1.07	-1.63	-1.75	-0.85	-1.41	-1.42
Panel C: Annualized real return						
5	2.80	1.79	1.64	3.12	2.13	2.01
10	1.03	0.28	0.40	1.31	0.59	0.69
20	0.72	0.29	0.40	0.90	0.50	0.58
Panel D: Annualized volatility						
5	32.88	35.22	30.12	30.89	33.30	28.18
10	18.08	20.33	17.32	16.63	18.81	15.88
20	8.65	9.99	8.59	7.83	9.08	7.78
Panel E: Sharpe ratio						
5	0.15	0.08	0.09	0.21	0.10	0.14
10	0.16	0.04	0.05	0.20	0.07	0.13
20	0.23	0.09	0.11	0.29	0.13	0.16

Note: The table reports the average optimal weight for stocks and bonds and statistics on ex-post performances of the dynamic investment strategies based on the unrestricted VAR(1) models. Return and volatility are in annualized percent.

Figure 1: Observable data

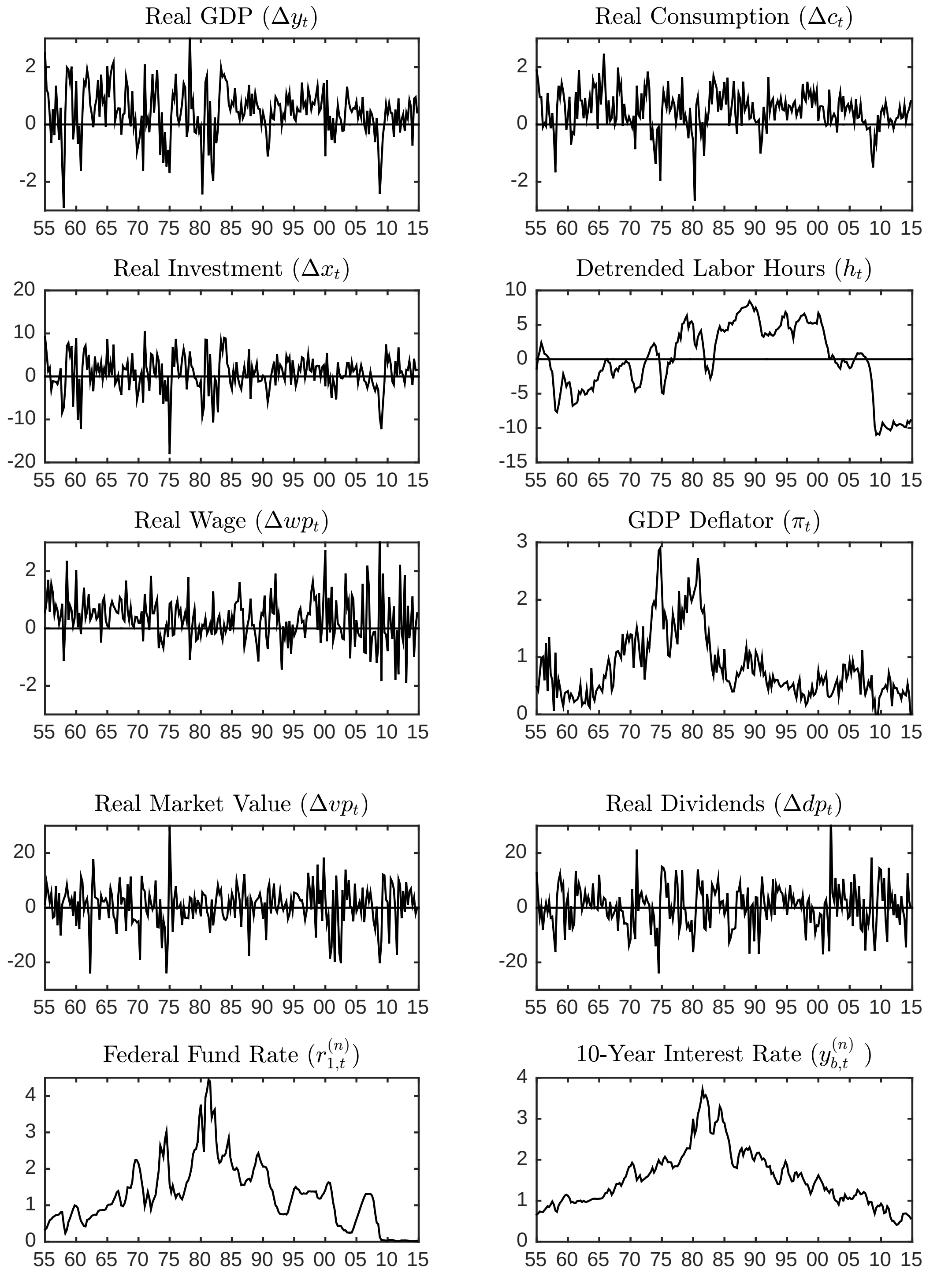
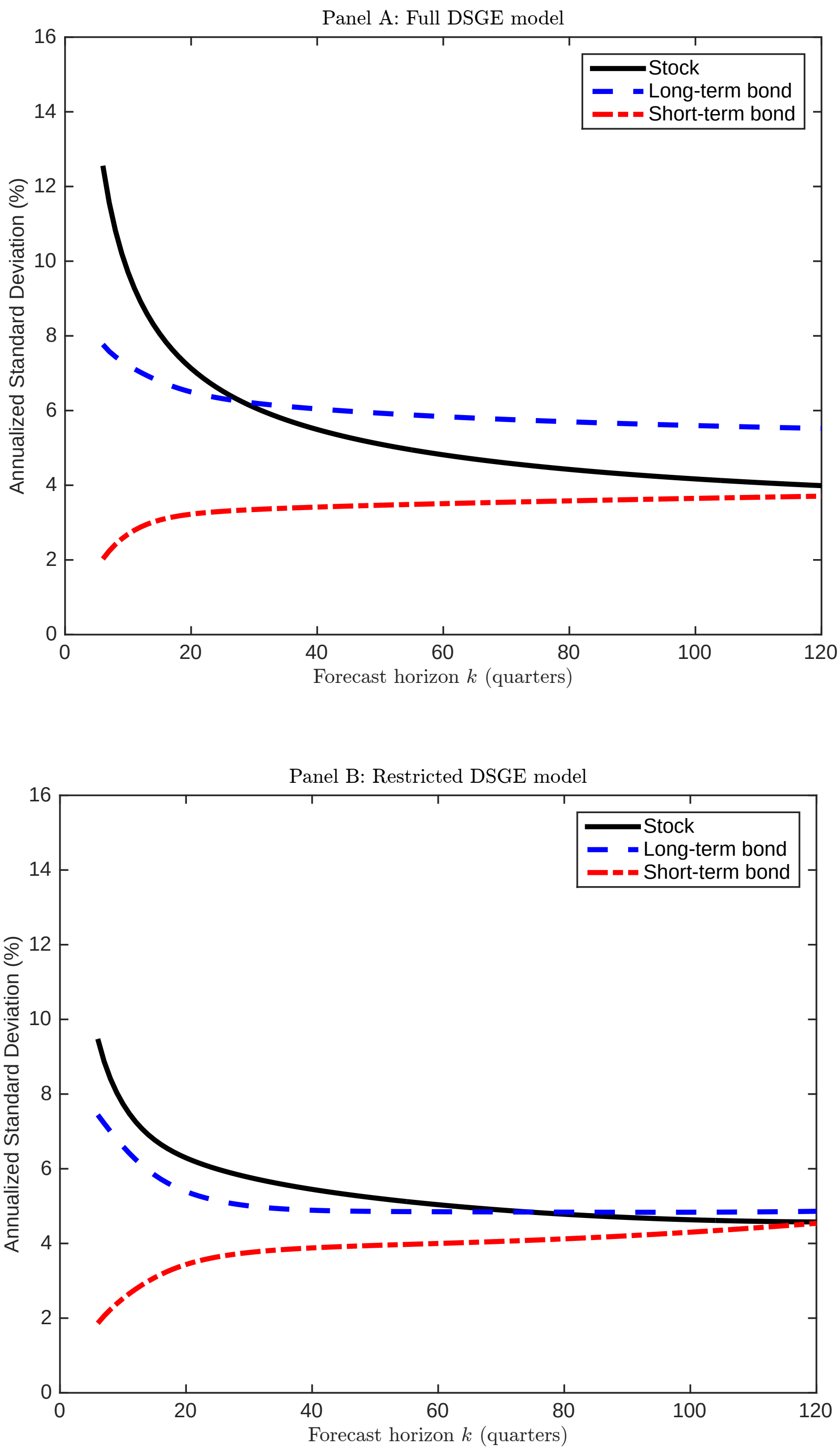
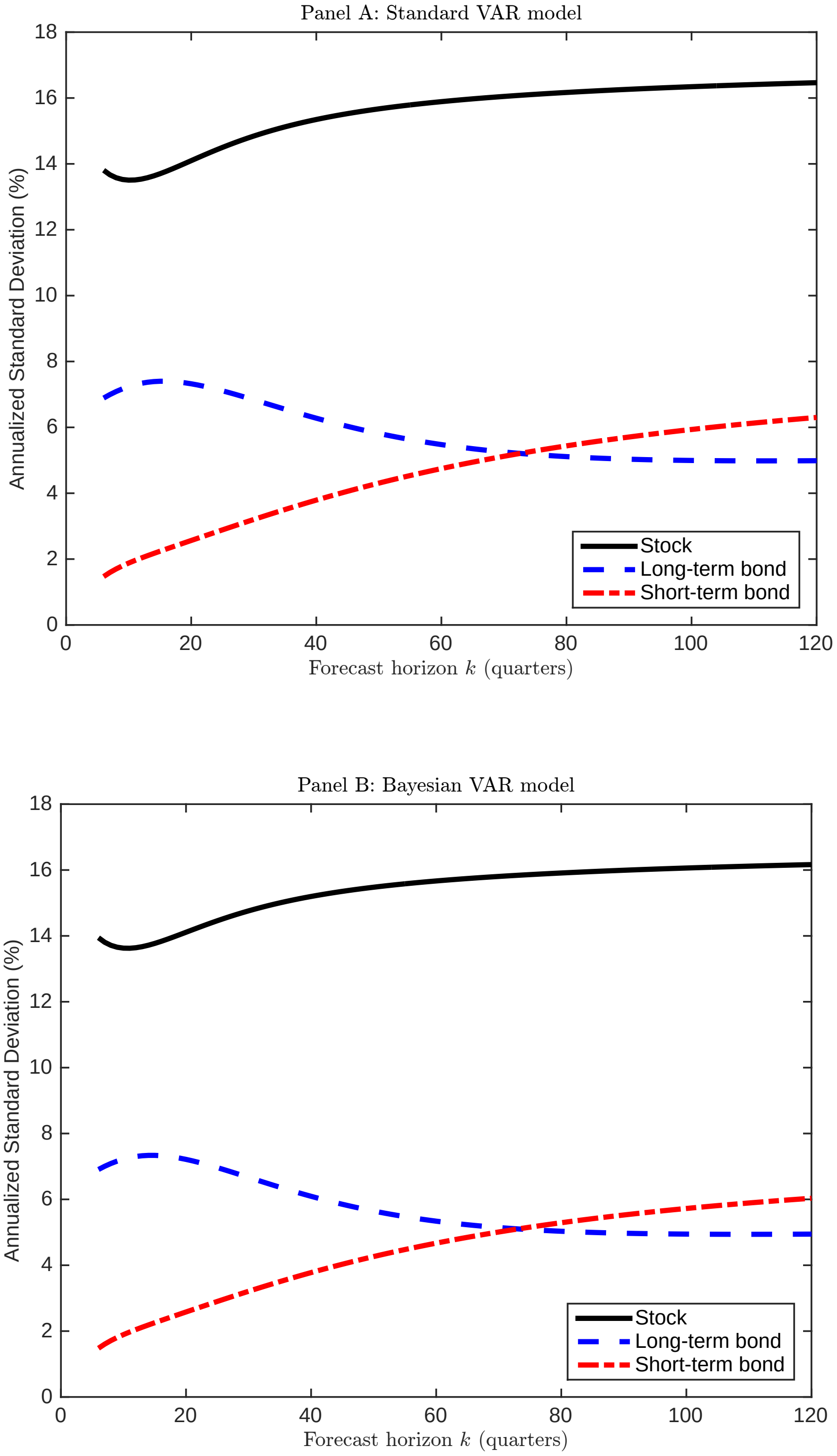


Figure 2: Annualized volatility of real returns as of 2014Q4 – DSGE models



Note: The figure displays the term structure of risk for 2014Q4 for the full DSGE model (Panel A) and the restricted DSGE model (Panel B).

Figure 3: Annualized volatility of real returns as of 2014Q4 – VAR models



Note: The figure displays the term structure of risk for 2014Q4 for the standard VAR and Bayesian VAR models.

Figure 4: Optimal portfolio weights – Full DSGE and standard VAR models

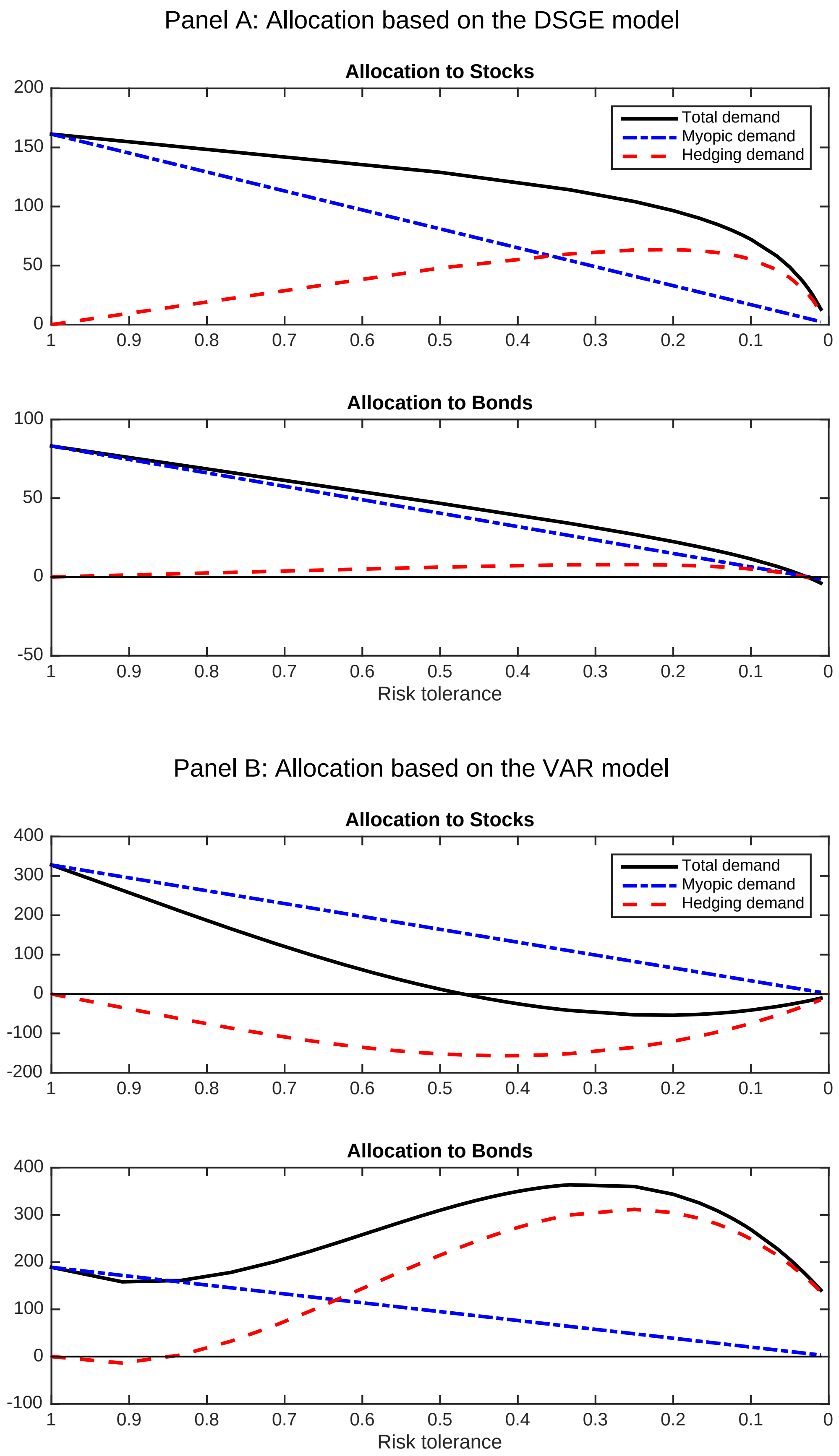
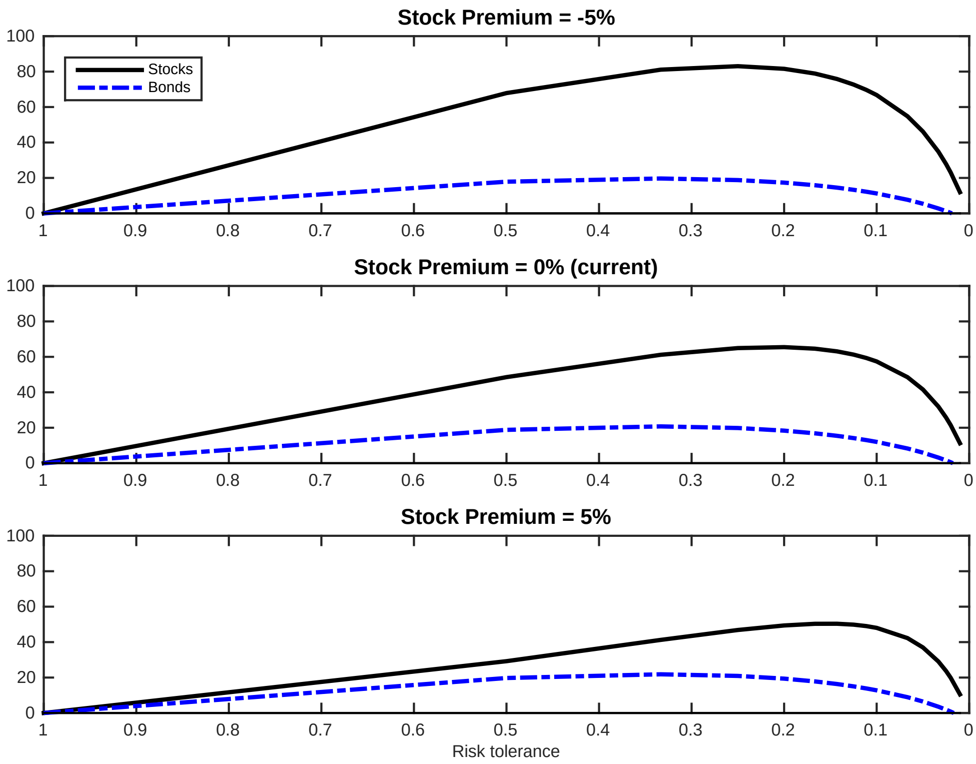


Figure 5: Hedging demands for the full DSGE model

Panel A: Change in the stock premium (bond premium at current value)



Panel B: Change in the bond premium (stock premium at current value)

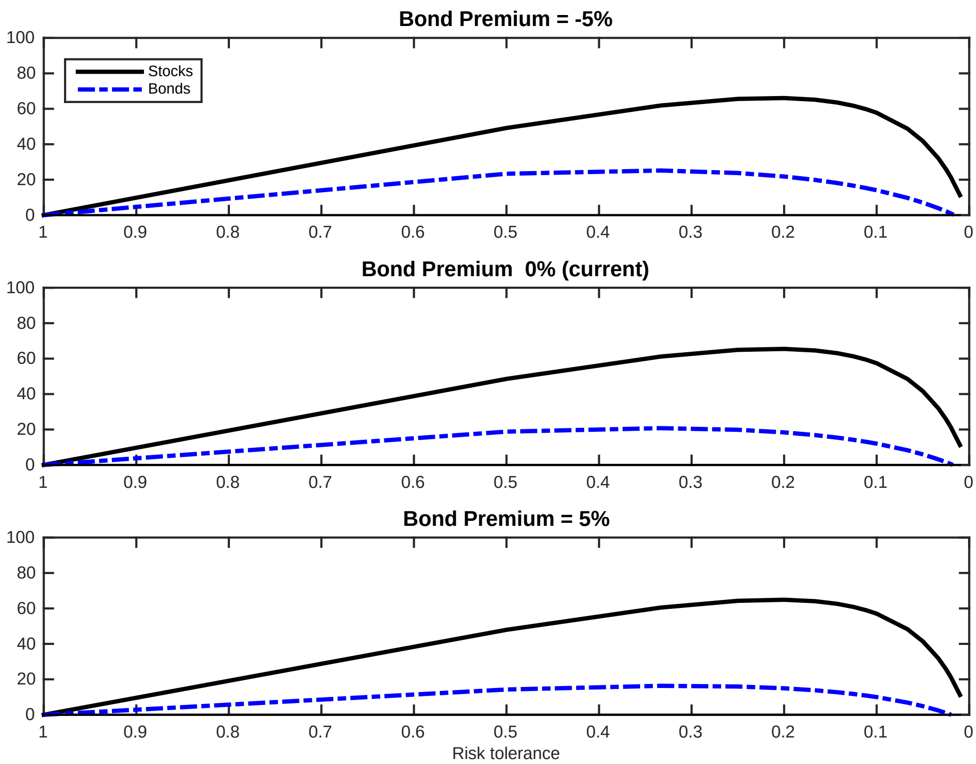


Figure 6: Average portfolio weights for a given date ($\gamma = 5$)

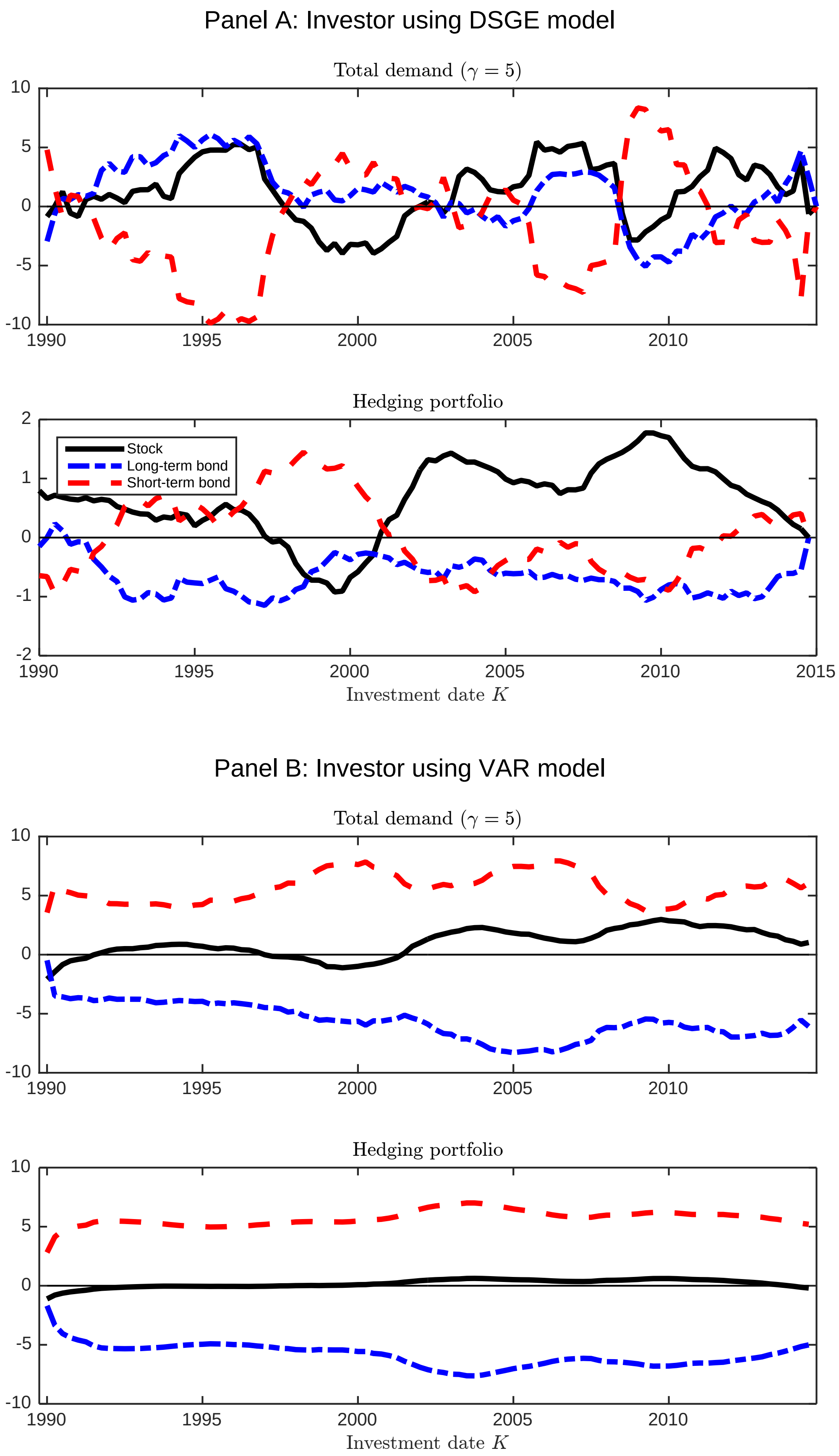


Figure 7: Average portfolio weights for a given date ($\gamma = 20$)

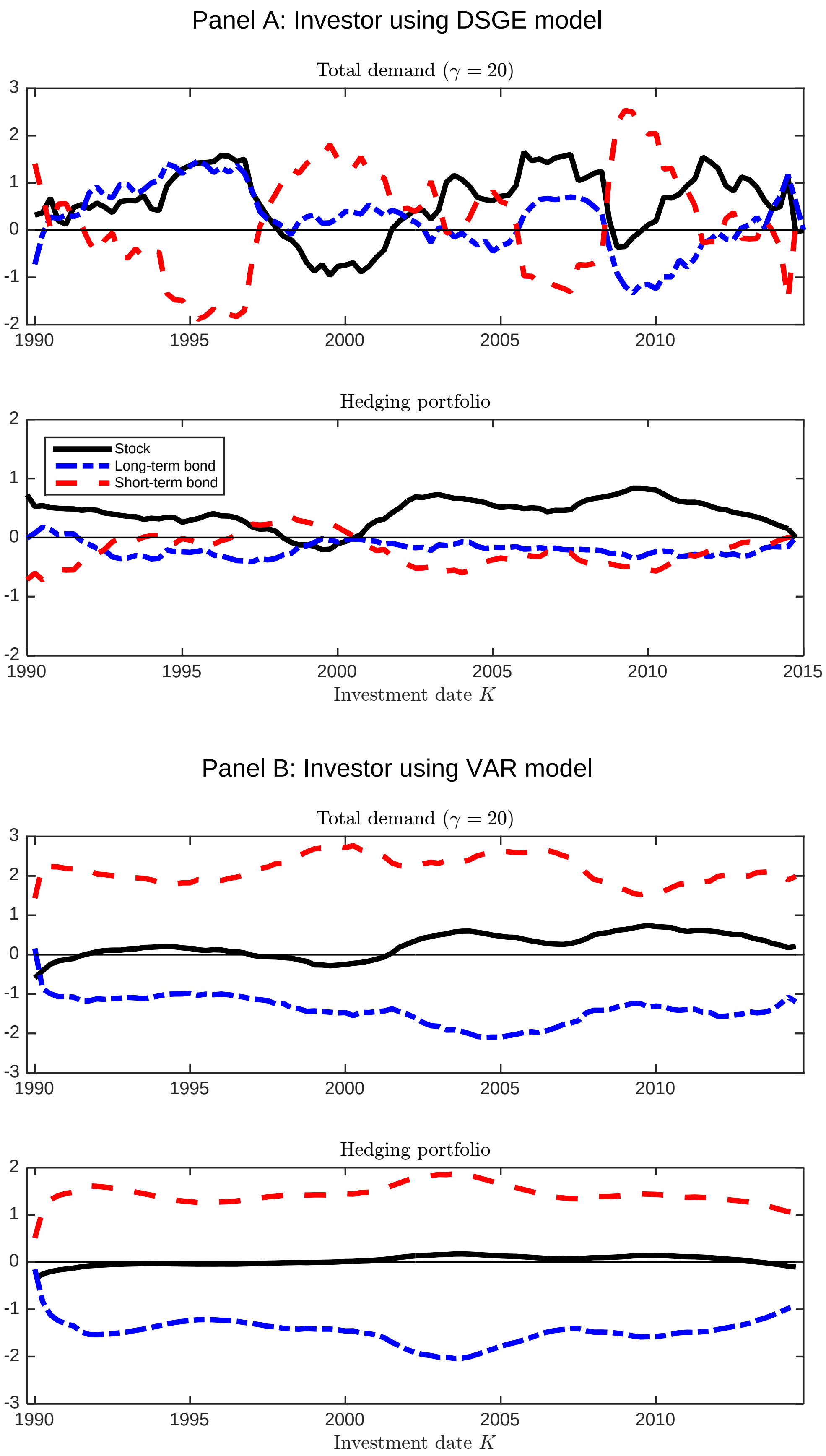
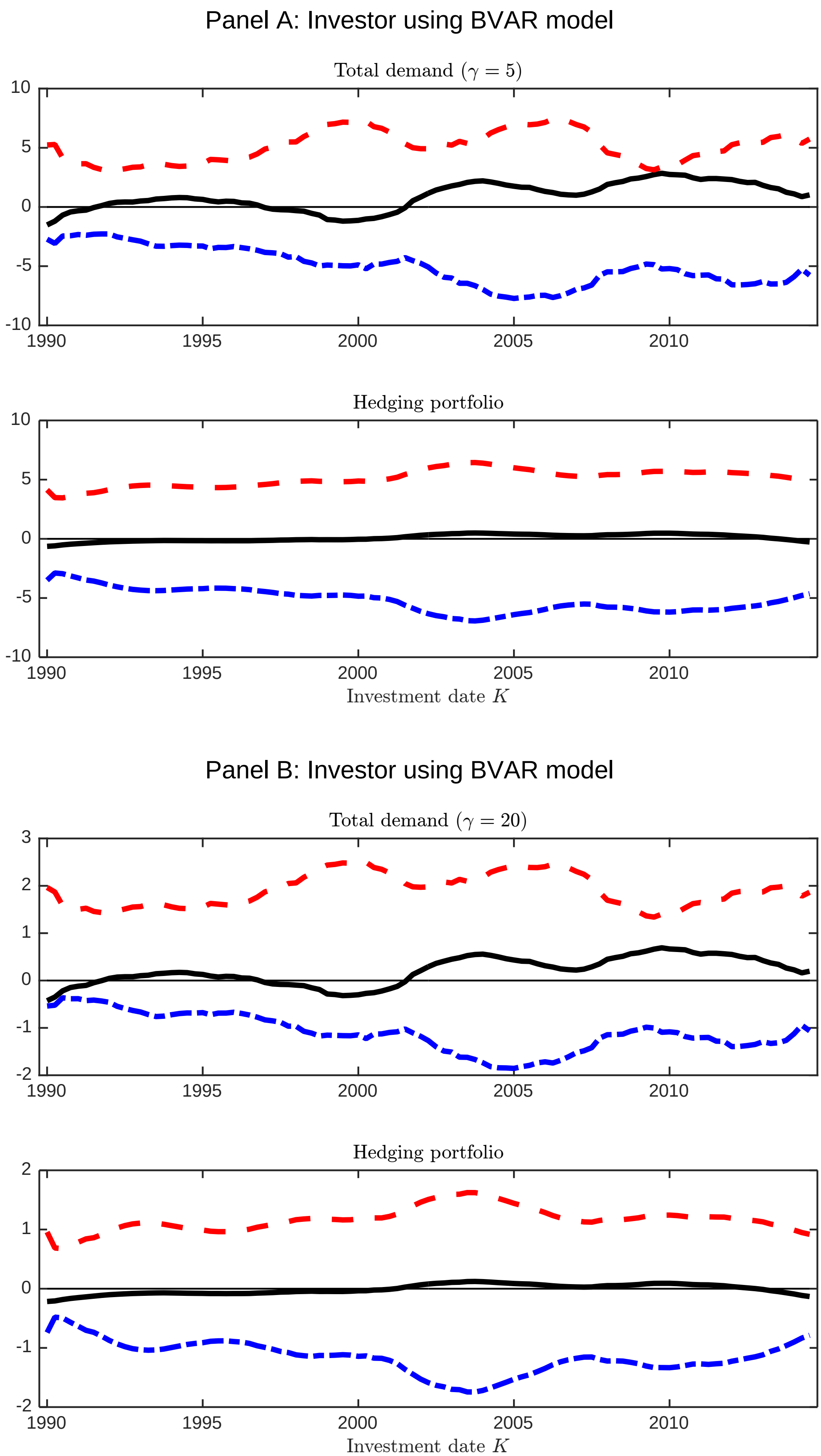


Figure 8: Average portfolio weights for a given date – Bayesian VAR model



Appendices

A Data

Per-capita variables are computed using the U.S. population over 16 years of age. Real variables are computed using the GDP deflator (NIPA Table 1.1.9). Observable variables are defined as follows:

- Per-capita real GDP ($\Delta \log(GDP_t)$) is the log-difference in GDP adjusted by population and inflation (from NIPA Table 1.15, line 1).
- Per-capita real consumption ($\Delta \log(CONS_t)$) is the log-difference in consumption adjusted by population and inflation (from NIPA Table 1.15, line 2).
- Per-capita real investment ($\Delta \log(INV_t)$) is the log-difference in investment adjusted by population and inflation (from NIPA Table 1.15, line 7).
- Per-capita labor hours ($\log(HRS_t)$) is the log of total labor hours. Labor hours are defined as total employment multiplied by the average workweek duration (from Current Employment Statistics).
- Real wage rate ($\Delta \log(WAGE_t)$) is the log-difference in hourly compensation rate in nonfarm business adjusted for inflation (from the Bureau of Labor Statistics, BLS Series id: PRS85006103).
- Inflation ($\Delta \log(P_t)$) is the log-difference in the GDP deflator.
- Per-capita real market value of nonfinancial firms ($\Delta \log(CAP_t)$) is computed as liabilities (line 21) minus financial assets (line 6) plus the market value of equities outstanding (line 35) per capita and in real terms (from Flow of Funds Account of the United States, Table B.102).
- Per-capita real dividends of nonfinancial firms ($\Delta \log(DIV_t)$). The series includes net buybacks and net financial acquisitions, minus the net increase in financial liabilities. It is computed as the net value added (line 19) minus compensation

of employees (line 20) minus taxes on production and imports less subsidies (line 23) minus net interest and miscellaneous payments (line 24) minus business current transfer payments (line 26) minus taxes on corporate income (line 28) (from NIPA, Table 1.14).

- The federal funds rate (FFR_t) (from the FRED database).
- The long-term interest rate (LTR_t) is the 10-year Treasury constant maturity yield (from the FRED database). Although long-term bonds are described as zero-coupon bonds in the model, the common investment vehicles are coupon bonds. Thus, we adapt the definition of the bond holding-period return to be consistent with that of coupon bonds.It is defined as:

$$r_{b,t+1} = D_t \; y_{b,t} - (D_t - 1) \; y_{b,t+1},$$

where D_t is Macaulay's duration defined as:

$$D_t = \frac{1 - e^{-10 y_{b,t}}}{1 - e^{-y_{b,t}}} \simeq \frac{1 - (1 + LTR_t)^{-10}}{1 - (1 - LTR_t)^{-1}}.$$

B DSGE Model

B.1 Description of the Model

We follow the presentation of the model proposed by Smets and Wouters (2007) and Alpanda (2013) and mostly focus on the new aspects of our model in this framework, i.e., the introduction of a complete term structure and portfolio adjustment frictions.

B.1.1 Labor Intermediaries

Labor intermediaries hire the labor services of households, aggregate them, and offer a composite labor service, h_t, to intermediate good producers. The labor service supplied by household j is denoted by $h_t^s(j)$. The composite labor service is aggregated by using a Dixit-Stiglitz aggregator (Dixit and Stiglitz, 1977):

$$h_t = \left[\int_0^1 h_t^s(j)^{(\Psi_t - 1)/\Psi_t} \, dj \right]^{\Psi_t/(\Psi_t - 1)}, \tag{A.1}$$

where $\psi_t = \Psi_t/(\Psi_t - 1)$ is a wage markup shock.[13] At the steady state, $\psi = \Psi/(\Psi - 1)$ is the gross markup of real wages received by households over the marginal rate of substitution between consumption and leisure. The aggregate labor services are then sold to the intermediate good producers. Maximization of the labor intermediaries' profit, $W_t h_t - \int_0^1 W_t(j) h_t^s(j) dj$, gives the labor demand curve of household j:

$$h_t^s(j) = \left[\frac{W_t(j)}{W_t}\right]^{-\Psi_t} h_t, \tag{A.2}$$

where W_t is the aggregate nominal wage.

B.1.2 Households

The economy is populated with infinitely lived households. The population of each household is denoted by N_t, which grows as $N_{t+1} = \eta N_t$. As in Fuhrer (2000), households have consumption habits, such that their utility depends on their current consumption relative to the past aggregate consumption. The habit level of consumption is defined as ζC_{t-1}, where C_{t-1} is the past level of aggregate consumption and ζ is the habit parameter. Each household j maximizes the expected utility defined over surplus consumption, $C_t(j) - \zeta C_{t-1}$, and labor supply, $h_t^s(j)$:

$$E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau} U_t(j) N_t, \tag{A.3}$$

with the following period utility:

$$U_t(j) = v_t \left[\frac{(C_t(j) - \zeta C_{t-1})^{1-\sigma_C}}{1-\sigma_C}\right] \exp\left(-\xi \frac{1-\sigma_C}{1+\sigma_L} \left(h_t^s(j)\right)^{1+\sigma_L}\right), \tag{A.4}$$

where β is the time discount factor ($\beta < 1$), v_t is the preference shock that affects the discount rate, σ_C is the inverse of the elasticity of intertemporal substitution (and relative risk aversion), σ_L is the inverse of the elasticity of the labor supply, and ξ is a level parameter, which is set such that labor equals 1 at the steady state of the model.[14]

[13] The dynamics of the shocks and risk premia are fully described in Section ??.

[14] Rudebusch and Swanson (2012) investigate a DSGE model with Epstein-Zin preferences and long-term bond risk instead of habit formation. They find that the fit of risk premia is improved, without deteriorating the fit of the key macro variables.

Households own the intermediate good firms and trade shares of these firms. Household j holds $S_t(i,j)$ shares of intermediate firm i and receives $D_t(i)$ as per-share dividends. The value of a share of firm i is $V_t(i)$. In contrast to Smets and Wouters (2007) and Alpanda (2013), we also allow households to carry a portfolio of nominal zero-coupon government bonds with remaining maturities ranging from 1 to K periods. Household j holds $Q_t^{(k)}(j)$ bonds of maturity k at time t, which pay 1 dollar at the end of period $t+k-1$. The price of such a bond is $B_t^{(k)}$, where $B_t^{(0)}=1$. This extension to a model in which households can hold stocks and bonds is an important contribution of our paper, as it will allow us to analyze long-term investment strategies in cash, bonds, and stocks, along the lines of Campbell et al. (2003), within a DSGE framework.

In introducing time variability in the risk premia, we follow the approach proposed by Marzo et al. (2008) and Falagiarda and Massimiliano (2012), who describe bond market segmentation through portfolio adjustment frictions. Given these frictions, which we also interpret as rebalancing costs, households have a preference for holding bonds of different maturities, resulting in nonzero demands for the various maturities. We denote the rebalancing costs for the equity and bond holdings by $\Phi_{s,t}$ and $\Phi_{b,t}^{(k)}$, $k=1,\cdots,K$, respectively, where $\Phi_{b,t}^{(1)}$ is normalized to 0. As we will show later, these rebalancing costs can be interpreted as time-varying risk premia for the various risky assets. The budget constraint of household j in period t is:

$$
\begin{aligned}
&N_t C_t(j) + \sum_{k=1}^{K-1}(1+\Phi_{b,t}^{(k)})\frac{B_t^{(k)}}{P_t}(Q_t^{(k)}(j) - Q_{t-1}^{(k+1)}(j)) + (1+\Phi_{b,t}^{(K)})\frac{B_t^{(K)}}{P_t}Q_t^{(K)}(j) \\
&+(1+\Phi_{s,t})\int_0^1 \frac{1}{P_t}[V_t(i)S_t(i,j) - (V_t(i)+(1-\tau_d)D_t(i))S_{t-1}(i,j)]di + \Phi_{w,t}(j) \\
&\leq (1-\tau_h)\frac{W_t(j)}{P_t}N_t h_t^s(j) + \frac{Q_{t-1}^{(1)}(j)}{P_t} - \frac{T_t}{P_t}, \qquad \text{(A.5)}
\end{aligned}
$$

where P_t is the aggregate price, τ_h is the tax on labor, τ_d is the tax on dividend income, and T_t is a lump-sum tax (see McGrattan and Prescott, 2005). We assume that after-tax dividends are instantaneously reinvested into the shares of the firms at no cost, until the household reoptimizes its portfolio allocation. As in Rotemberg (1983) and Chugh (2006), wage stickiness is introduced in the form of a quadratic adjustment cost, $\Phi_{w,t}(j)$, defined as follows:

$$\Phi_{w,t}(j) = \frac{\kappa_w}{2}(\Psi - 1)(1-\tau_h)\left[\frac{W_t(j)/W_{t-1}(j)}{(\pi\gamma)(\pi_{t-1}/\pi)^{\eta_w}} - 1\right]^2 \frac{W_t}{P_t}N_t h_t,$$

where κ_w is the cost-of-adjustment parameter; η_w is the indexation parameter of wage adjustments to past aggregate inflation, denoted by $\pi_{t-1} = P_{t-1}/P_{t-2}$; and π is the steady-state inflation rate.

Household j maximizes expected utility (A.3) subject to the sequence of budget constraints (A.5) for $t = \tau, \cdots, \infty$. At the symmetric equilibrium, the first-order condition with respect to consumption yields the following marginal utility of consumption, i.e., the Lagrange multiplier with respect to the household budget constraint:

$$\Lambda_t = \upsilon_t (C_t - \zeta C_{t-1})^{-\sigma_C} \exp\left(-\xi \frac{1-\sigma_C}{1+\sigma_L}(h_t^s)^{1+\sigma_L}\right).$$

B.1.3 Final Good Producers

Final good producers purchase goods from intermediate firms, aggregate them, and sell the final good to consumers. The composite final good is aggregated by using a Dixit-Stiglitz aggregator:

$$Y_t = \left[\int_0^1 Y_t(i)^{(\Theta_t-1)/\Theta_t}\, di\right]^{\Theta_t/(\Theta_t-1)},$$

where $\theta_t = \Theta_t/(\Theta_t - 1)$ is a price markup shock. At the steady state, $\theta = \Theta/(\Theta - 1)$ is the price gross markup over the marginal cost. Maximization of the final producers' profit, $P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$, yields the demand curve for the intermediate goods:

$$Y_t(i) = \left[\frac{P_t(i)}{P_t}\right]^{-\Theta_t} Y_t. \tag{A.6}$$

B.1.4 Intermediate Good Producers

Intermediate good producers own capital stock and are price setters in the goods market. The production function of intermediate good producer i is:

$$Y_t(i) = z_t \left[u_t(i) K_{t-1}(i)\right]^{\alpha} \left[A_t N_t h_t(i)\right]^{1-\alpha} - (\eta\gamma)^t f,$$

where z_t is the aggregate technology shock, $K_t(i)$ is the capital owned by firm i, $u_t(i)$ is the utilization rate of capital, $N_t h_t(i)$ is the amount of labor that is used in the production of intermediate good i, A_t is the trend of productivity growth ($A_t = \gamma^t$), and $(\eta\gamma)^t f$ is the fixed cost of production.[15] Parameter α represents the share of capital in production.

Capital accumulation is given by:

$$K_t(i) = (1-\delta)K_{t-1}(i) + \left[1 - \frac{\kappa_I}{2}\left(\frac{I_t(i)}{(\eta\gamma)I_{t-1}(i)} - 1\right)^2\right] z_t^I I_t(i),$$

where $I_t(i)$ is the investment of firm i, z_t^I is an investment-specific technology shock, δ is the depreciation rate of capital, and κ_I is the cost-of-investment-adjustment parameter. The term in squared brackets captures the cost of investment adjustment.

Dividends that are paid out to shareholders are equal to the residual of the total revenue after payments for wages, investments, price adjustment costs, and taxes are subtracted:

$$\begin{aligned}\frac{D_t(i)}{P_t} &= (1-\tau_s)\frac{P_t(i)}{P_t}Y_t(i) - \frac{W_t}{P_t}N_t h_t(i) - I_t(i) - \Phi_{p,t}(i)\\ &\quad -\tau_y\left[(1-\tau_s)\frac{P_t(i)}{P_t}Y_t(i) - \frac{W_t}{P_t}N_t h_t(i) - \delta_a K_{t-1}(i)\right] + (\eta\gamma)^t\Phi_{d,t},\end{aligned}$$

where τ_s and τ_y are proportional taxes on sales and income, respectively, and δ_a is the accounting depreciation rate. Regarding wages, we introduce price stickiness in the form of quadratic adjustment cost (Rotemberg, 1983, and Chugh, 2006). The quadratic cost of price adjustment, $\Phi_{p,t}$, is defined as follows:

$$\Phi_{p,t}(i) = \frac{\kappa_p}{2}(\Theta - 1)(1-\tau_s)(1-\tau_y)\left[\frac{P_t(i)/P_{t-1}(i)}{\pi(\pi_{t-1}/\pi)^{\eta_p}} - 1\right]^2 Y_t,$$

where κ_p is the cost-of-adjustment parameter and η_p is the price indexation parameter. $(\eta\gamma)^t\Phi_{d,t}$ is an exogenous transfer from the government to firms, where the stationary component $\Phi_{d,t}$ represents a dividend shock.

The objective of intermediate good producers is to maximize after-tax dividends:

$$E_\tau \sum_{t=\tau}^{\infty} \beta^{t-\tau}\frac{\Lambda_t}{\Lambda\tau}(1-\tau_d)\left[\frac{D_t(i)}{P_t} - \frac{\kappa_u}{1+\chi}\left(u_t(i)^{1+\chi} - 1\right)K_{t-1}(i)\right],$$

[15] The fixed cost is set such that intermediate good producers make no economic profit in the long run.

where κ_u is a scale parameter ensuring that the utilization rate equals 1 at the steady state and χ is a capacity utilization elasticity parameter. The last term measures the cost of capital utilization.

B.1.5 Government and the Central Bank

As in Smets and Wouters (2007), government expenditure is defined as $G_t = (\eta\gamma)^t g_t$, the stochastic component of which, g_t, responds to productivity innovations. At time t, the government issues new bonds with maturities $k = 1, \cdots, K$ and reimburses the bonds issued $k = 1, \cdots, K$ periods beforehand. The budget constraint in period t is given by:

$$\begin{aligned} G_t + (\eta\gamma)^t \Phi_{d,t} + \sum_{k=1}^{K} \frac{Q_{t-k}^{(k)}}{P_t} &= \frac{T_t}{P_t} + \tau_h \frac{W_t}{P_t} N_t h_t + \tau_d \frac{D_t}{P_t} S_{t-1} + \tau_s Y_t \\ &+ \tau_y \left[(1-\tau_s) Y_t - \frac{W_t}{P_t} N_t h_t - \delta_a K_{t-1} \right] + \sum_{k=1}^{K} \frac{B_t^{(k)} Q_t^{(k)}}{P_t}. \end{aligned}$$

We assume that the lump-sum tax, T_t, paid by households reacts to the debt level to avoid an explosive path of debt.

The central bank's Taylor rule gives the dynamics of the one-period gross nominal interest rate:

$$\frac{R_{1,t}^{(n)}}{R_1^{(n)}} = \left(\frac{R_{1,t-1}^{(n)}}{R_1^{(n)}} \right)^{\rho_r} \left[\left(\frac{\pi_t}{\bar{\pi}_t} \right)^{a_\pi} \left(\frac{Y_t}{Y_t^n} \right)^{a_y} \left(\frac{Y_t / Y_{t-1}}{Y_t^n / Y_{t-1}^n} \right)^{a_g} \right]^{1-\rho_r} \epsilon_{r,t},$$

where $R_1^{(n)}$ is the steady-state level of the nominal policy rate, ρ_r is the interest rate smoothing parameter, and a_π, a_y, and a_g are the Taylor rule's weights. Y_t^n is the natural rate of output, which is defined as the level of output that would prevail under flexible prices in the absence of cost-push shocks.[16] $\epsilon_{r,t}$ is the monetary policy shock, and $\bar{\pi}_t$ is the time-varying inflation rate targeted by the central bank, which is described as an ARMA(1,1) process:

$$\log \bar{\pi}_t = \rho_{\bar{\pi}} \log \bar{\pi}_{t-1} + \eta_{\bar{\pi},t} - \varsigma_{\bar{\pi}} \eta_{\bar{\pi},t-1}. \tag{A.7}$$

[16]The natural rate of output is computed in a model-consistent manner by solving the model with flexible prices and wages. See Smets and Wouters (2003).

B.1.6 Market-Clearing Conditions

At equilibrium, all the markets clear, which results in the following relations:

- Goods market-clearing condition:

$$N_t C_t + I_t + G_t = Y_t - \Phi_{w,t} - \Phi_{p,t} - \Phi_{ac,t},$$

  where $\Phi_{ac,t}$ denotes the sum of the adjustment costs for bonds and stocks paid by households, which represent a resource cost.

- Labor services market-clearing condition:

$$h_t = \int_0^1 h_t(i)\, di = \left[\int_0^1 h_t^s(j)^{1/\psi_t}\, dj \right]^{\psi_t}.$$

- Bond market-clearing condition:

$$\bar{Q}_t^{(k)} = \int_0^1 Q_t^{(k)}(j)\, dj, \qquad \forall k = 1, \cdots, K,$$

  where $\bar{Q}_t^{(k)} = \sum_{\tau=0}^{K-k} Q_{t-\tau}^{(\tau+k)}$ denotes the number of government bonds with remaining maturity k available at date t to households.

- Equity market-clearing condition:

$$S_t(i) = \int_0^1 S_t(i,j)\, dj, \qquad \forall i.$$

Equilibrium is attained when all agents maximize their objective functions and all markets clear. We assume a symmetric equilibrium, where all the households and all the intermediate good producers have the same characteristics.

B.1.7 Financial Asset Returns

Under complete markets and in the absence of arbitrage opportunities, the first-order conditions with respect to asset holdings yield the pricing equations for the financial assets. For a one-period bond, we have:

$$1 = \beta\ E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \frac{R^{(n)}_{1,t+1}}{\pi_{t+1}} \right],$$

where $R^{(n)}_{1,t+1} = 1/B^{(1)}_t$ is the one-period gross nominal interest rate set by the central bank for the period between t and $t+1$.

For longer-maturity bonds, we have:

$$1 = \beta\ E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \frac{1+\Phi^{(k-1)}_{b,t+1}}{1+\Phi^{(k)}_{b,t}} \frac{R^{(k)(n)}_{b,t+1}}{\pi_{t+1}} \right], \text{ for } k = 2, \cdots, K,$$

where $R^{(k)(n)}_{b,t+1} = B^{(k-1)}_{t+1}/B^{(k)}_t$ denotes the gross nominal holding period return of the bond of maturity k held between t and $t+1$. We also define $Y^{(k)(n)}_{b,t} = (B^{(k)}_t)^{1/k}$ the gross nominal yield to maturity of a k-period bond issued at time t.

For a given stock i, the pricing equation is (in the following expressions, we omit the index i of the firm):

$$1 = \beta\ E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \frac{1+\Phi_{s,t+1}}{1+\Phi_{s,t}} \frac{R^{(n)}_{s,t+1}}{\pi_{t+1}} \right].$$

We define $R^{(n)}_{s,t+1} = (V_{t+1} + (1-\tau_d)D_{t+1})/V_t$ as the gross nominal return of a stock held between t and $t+1$.

B.2 Parametrization and Estimation

Some of the parameters of the model are calibrated when their values can be deduced from accounting data, fiscal data, or long-term trend data. For this calibration, we follow the approach of Alpanda (2013) by computing the parameters over the sample period. Their values are reported in Table A1. The average real growth is equal to 3.1% per year (population growth, η, of 1.4% and per-capita output growth, γ, of 1.7%). The average annual inflation and short-term rates are 3.3% and 5.1%, respectively. Tax rates and shares are estimated by using accounting data to match steady-state relations over the postwar period.

To facilitate comparison with previous work, we redefine some parameters. The capacity utilization elasticity is defined as $\chi^e = \chi/(1+\chi)$. Similarly, we rescale the adjustment costs as follows:

$$\kappa_p = \frac{10(\theta-1)+1}{(1-\kappa_p^e)(1-\tilde{\beta}\kappa_p^e)}\kappa_p^e, \quad \text{and} \quad \kappa_w = \frac{10(\psi-1)+1}{(1-\kappa_w^e)(1-\tilde{\beta}\kappa_w^e)}\kappa_w^e.$$

We also set $\xi = ((1-\tau_h)\bar{w}p/\bar{y})(\psi(1-\zeta/\gamma)\bar{c}/\bar{y})$ and $f = \bar{y}(\theta-1)$ to normalize the labor supply and intermediate good producers' profit in the long run.

[Insert Table A1 here]

The model is estimated by using Bayesian methodology.[17] The dynamic system is mapped to a state-space representation for the set of observable variables. The Kalman filter is then used to evaluate the likelihood of the observed variables and to form the posterior distribution of the structural parameters by combining the likelihood function with a joint density characterizing some prior beliefs. Given the specification of the model, the posterior distribution cannot be recovered analytically but can be evaluated numerically by using a Markov Chain Monte-Carlo sampling approach. More specifically, we rely on the Metropolis-Hastings (MH) algorithm to obtain random draws from the posterior distribution of the parameters.[18]

Because Bayesian estimation requires some priors on the parameters, we begin with a description of these priors, reported in Tables A2 and A3. Our priors are rather similar to, although generally less restrictive than, those adopted in previous studies (in particular, Smets and Wouters, 2003; Jondeau and Sahuc, 2008) and closely match the priors selected by Alpanda (2013). We assume a Beta distribution for the following parameters bounded between zero and one: the habit persistence parameter (ζ), the degree of price and wage indexation (η_p and η_w), the smoothing parameter in the monetary policy rule (ρ_r), the adjustment cost parameters (κ_w^e, κ_ρ^e, and κ_I^e). The inverse of the consumption elasticity of substitution (σ_C), the inverse of the elasticity of labor supply (σ_L), the price and wage markups (θ and ψ), and the Taylor rule parameters (a_π, a_y, and a_g) have a normal prior.

[17] For the estimation of DSGE models, Schorfheide (2003), Fernandez-Villaverde and Rubio-Ramirez (2004), and Smets and Wouters (2007), among others, propose the use of Bayesian methodology.

[18] We simulate two blocks of 250,000 random draws. The first 20,000 observations are discarded to eliminate any dependence on the initial values. The mode and Hessian of the posterior distribution evaluated at the mode are used to initialize the MH algorithm.

The autoregressive and moving-average parameters for shocks have a Beta distribution with a mean value of 0.5 and a standard deviation of 0.2. The cross-correlation parameters describing the dynamics of the bond and stock premia have a normal distribution with a mean value of 0 and a standard deviation of 0.5. All priors on the shock variances have an inverse gamma distribution.

The mean and confidence interval of the posterior distribution of the parameters are also reported in the table. Regarding household behavior, our estimate of the inverse of the consumption elasticity of substitution (σ_C) is 2.85, while the inverse of the elasticity of labor disutility (σ_L) is approximately 1.64. The habit persistence parameter ζ is estimated to be 0.73. Regarding the behavior of firms, we find that wage indexation is significantly lower than price indexation ($\eta_w = 0.34$ versus $\eta_p = 0.64$). In contrast,the adjustment costs is much higher for wages than for prices ($\kappa_w^e = 0.8$ versus $\kappa_p^e = 0.2$). In the reaction function, the long-run impact of inflation and the output gap on the short-term interest rate is approximately 1.04 and 0.13, respectively. Our parameter estimates are broadly in line with the estimates reported by Alpanda (2013), which are shown in the last column of our table for convenience.[19] The new parameters correspond to the dynamics of the inflation target and the bond and stock premia.

[Insert Tables A2 and A3 here]

[19]The model and sample period in Smets and Wouters (2007) differ from ours. In particular, their wage and inflation dynamics are modeled within a staggered price setting à la Calvo (1983), whereas we introduce stickiness through quadratic adjustment costs à la Rotemberg (1983). For these reasons, our parameter estimates cannot be directly compared with those reported in Smets and Wouters (2007).

Table A1: Value of the calibrated parameters

	Symbol	Value
Average quarterly growth of the population	η	1.0035
Average quarterly growth of per-capita output	γ	1.0042
Trend inflation factor	π	1.0086
Average (gross) nominal interest rate	$R_1^{(n)}$	1.0134
Adjusted time-discount factor	$\tilde{\beta}$	0.9935
Capital share parameter	α	28.37%
Depreciation rate of capital	δ	1.28%
Accounting depreciation rate	δ_a	1.40%
Tax rate on firm income	τ_y	31.5%
Tax rate on dividend income	τ_d	21.5%
Tax rate on sales	τ_s	8.9%
Tax rate on labor income	τ_h	35%
Share of consumption	$\bar{c}/\bar{y}$	65.0%
Share of government expenditure	$\bar{g}/\bar{y}$	19.1%
Share of investment	$\bar{\iota}/\bar{y}$	16.0%
Share of dividends	$\bar{dp}/\bar{y}$	5.2%
Share of labor compensation in total income	$\bar{wp}/\bar{y}$	65.3%

Note: The table reports the value of the calibrated parameters. These values are drawn from Alpanda (2013). The calibration is based on the data available over the sample period.

Table A2: Parameter estimates of the DSGE model

		Prior			Posterior distribution			Alpanda
		Dist.	Par.1	Par.2	Mean	5%	95%	(2013)
ζ	Habit	beta	0.7	0.1	0.728	0.718	0.737	0.954
σ_C	Consumption elasticity	norm	1.5	0.37	2.848	2.760	2.933	0.995
σ_L	Labor supply elasticity	norm	2	0.75	1.637	1.591	1.681	2.497
ψ	Wage Mark-up	norm	1.5	0.12	1.591	1.579	1.605	1.609
θ	Price Mark-up	norm	1.5	0.12	1.662	1.653	1.675	1.707
χ^e	Utilization elasticity	beta	0.5	0.15	0.476	0.466	0.486	0.187
κ_w^e	Adjustment cost - Wage	beta	0.5	0.1	0.799	0.788	0.811	0.863
κ_p^e	Adjustment cost - Price	beta	0.5	0.1	0.199	0.188	0.206	0.782
κ_I	Adjustment cost - Invest.	norm	4	1.5	7.106	7.011	7.222	4.473
η_w	Wage indexation	beta	0.5	0.15	0.336	0.330	0.343	0.518
η_p	Price indexation	beta	0.5	0.15	0.638	0.627	0.648	0.200
ρ_r	Taylor - smoothing	beta	0.75	0.1	0.825	0.815	0.835	0.819
a_π	Taylor - inflation	norm	1.5	0.25	1.039	1.029	1.053	1.449
a_y	Taylor - output gap	norm	0.12	0.05	0.128	0.125	0.131	0.071
a_g	Taylor - output growth	norm	0.12	0.05	0.210	0.206	0.215	0.236
ρ_v	AR term. Preference	beta	0.5	0.2	0.797	0.775	0.822	0.511
ρ_ψ	AR term. Wage mark-up	beta	0.6	0.2	0.913	0.894	0.931	0.864
ρ_θ	AR term. Price mark-up	beta	0.5	0.2	0.929	0.924	0.933	0.941
ρ_z	AR term. Technology	beta	0.5	0.2	0.989	0.988	0.991	0.974
ρ_I	AR term. Investment	beta	0.6	0.2	0.823	0.810	0.837	0.996
ρ_g	AR term. Government	beta	0.5	0.2	0.985	0.981	0.989	0.985
ρ_r	AR term. Monetary	beta	0.5	0.2	0.786	0.765	0.804	0.579
ρ_d	AR term. Dividend	beta	0.5	0.2	0.953	0.945	0.959	0.947
ρ_b	AR term. Bond risk	beta	0.5	0.2	0.341	0.318	0.363	–
ρ_s	AR term. Stock risk	beta	0.5	0.2	0.618	0.593	0.639	0.824
$\rho_{\bar{\pi}}$	AR term. Target inflation	beta	0.5	0.2	0.200	0.188	0.214	–
$\rho_{g,z}$	Cross-corr. Gvt-Prod.	beta	0.5	0.2	-0.270	-0.301	-0.240	0.685
ς_v	MA term. Preference	beta	0.5	0.2	0.562	0.550	0.575	0.460
ς_ψ	MA term. Wage mark-up	beta	0.5	0.2	0.859	0.816	0.897	0.802
ς_θ	MA term. Price mark-up	beta	0.5	0.2	0.386	0.371	0.402	0.893
ς_z	MA term. Technology	beta	0.5	0.2	0.724	0.710	0.740	0.063
ς_I	MA term. Investment	beta	0.5	0.2	0.291	0.266	0.318	0.927
ς_g	MA term. Government	beta	0.5	0.2	0.144	0.124	0.167	0.048
ς_r	MA term. Monetary	beta	0.5	0.2	0.225	0.212	0.234	0.428
ς_d	MA term. Dividend	beta	0.5	0.2	0.496	0.467	0.522	0.069
ς_b	MA term. Bond risk	beta	0.5	0.2	0.276	0.258	0.292	–
ς_s	MA term. Stock risk	beta	0.5	0.2	0.297	0.285	0.309	0.184
$\varsigma_{\bar{\pi}}$	MA term. Target inflation	beta	0.5	0.2	0.604	0.581	0.639	–

Table A3: Parameter estimates of the DSGE model (continued)

		Priors			Posterior distribution			Alpanda
		Dist.	Par.1	Par.2	Mean	5%	95%	(2013)
$\rho_{b,v}$	Cross-corr. Bond-Cons.	norm	0	0.5	0.042	0.002	0.092	–
$\rho_{b,\psi}$	Cross-corr. Bond-Wage	norm	0	0.5	-0.066	-0.122	-0.006	–
$\rho_{b,\theta}$	Cross-corr. Bond-Price	norm	0	0.5	0.068	0.023	0.116	–
$\rho_{b,z}$	Cross-corr. Bond-Techn.	norm	0	0.5	-0.103	-0.146	-0.078	–
$\rho_{b,I}$	Cross-corr. Bond-Invt	norm	0	0.5	-0.341	-0.371	-0.311	–
$\rho_{b,g}$	Cross-corr. Bond-Gvt	norm	0	0.5	-0.250	-0.299	-0.196	–
$\rho_{b,r}$	Cross-corr. Bond-Monetary	norm	0	0.5	0.328	0.275	0.382	–
$\rho_{b,d}$	Cross-corr. Bond-Dividend	norm	0	0.5	-1.071	-1.133	-0.994	–
$\rho_{b,s}$	Cross-corr. Bond-Stock	norm	0	0.5	0.663	0.607	0.734	–
$\rho_{s,v}$	Cross-corr. Stock-Cons.	norm	0	0.5	0.310	0.292	0.328	–
$\rho_{s,\psi}$	Cross-corr. Stock-Wage	norm	0	0.5	0.247	0.215	0.280	–
$\rho_{s,\theta}$	Cross-corr. Stock-Price	norm	0	0.5	1.143	1.091	1.180	–
$\rho_{s,z}$	Cross-corr. Stock-Techn.	norm	0	0.5	0.597	0.557	0.633	–
$\rho_{s,I}$	Cross-corr. Stock-Invt	norm	0	0.5	-0.797	-0.832	-0.755	–
$\rho_{s,g}$	Cross-corr. Stock-Gvt	norm	0	0.5	-0.536	-0.593	-0.485	–
$\rho_{s,r}$	Cross-corr. Stock-Monetary	norm	0	0.5	-0.204	-0.305	-0.111	–
$\rho_{s,d}$	Cross-corr. Stock-Dividend	norm	0	0.5	0.348	0.306	0.388	–
$\rho_{s,b}$	Cross-corr. Stock-Bond	norm	0	0.5	-0.180	-0.209	-0.150	–
Standard deviation of shocks ($\times$100)								
σ_v	Preference	invg	0.005	Inf	13.04	11.83	14.25	0.31$^{(*)}$
σ_ψ	Wage mark-up	invg	0.005	Inf	0.70	0.63	0.77	0.49
σ_θ	Price mark-up	invg	0.005	Inf	0.82	0.72	0.90	0.17
σ_z	Technology	invg	0.005	Inf	0.97	0.88	1.04	0.49
σ_I	Investment	invg	0.005	Inf	1.17	1.04	1.27	1.64
σ_g	Government	invg	0.005	Inf	1.96	1.80	2.11	1.93
σ_r	Monetary	invg	0.005	Inf	0.14	0.11	0.16	0.20
σ_d	Dividend	invg	0.005	Inf	1.17	1.07	1.27	0.79
σ_b	Bond risk premium	invg	0.005	Inf	7.62	6.47	8.65	–
σ_s	Stock risk premium	invg	0.005	Inf	3.22	2.93	3.54	1.42
$\sigma_{\bar{\pi}}$	Target inflation	invg	0.005	Inf	0.68	0.57	0.80	–

Note: The table reports the information about the prior and posterior distributions of the parameters. For the prior distribution, the table indicates the class of distribution and its two characteristic parameters. For the posterior distribution, the table reports the mean and the 5%–95% confidence interval. The acronyms "beta", "norm.", and "invg" stand for the beta, the normal, and the inverse gamma distributions. $^{(*)}$ We have rescaled consumption shock for readability purpose, so that its standard deviation is not comparable to the one reported by Alpanda (2003).

Table A4: Parameter estimates of the VAR model with h_t

	Δy_t	Δc_t	$\Delta \iota_t$	h_t	Δwp_t	π_t	$r_{1,t}$	$sp_{b,t}$	$x_{b,t}$	$\rho_{s,t}$	dpr_t	R^2
Panel A: Parameter estimates												
Δy_{t+1}	-0.32	0.57	0.07	-3.60	0.04	0.02	0.04	0.59	-0.01	0.02	0.00	0.29
	(2.04)	(4.47)	(2.55)	(1.64)	(0.59)	(0.13)	(0.43)	(3.14)	(0.80)	(2.80)	(0.44)	
Δc_{t+1}	-0.02	0.11	0.04	-4.37	0.09	-0.05	0.12	0.54	0.04	0.02	0.00	0.30
	(0.19)	(1.16)	(2.22)	(2.55)	(1.69)	(0.50)	(1.66)	(3.71)	(2.90)	(3.09)	(0.80)	
$\Delta \iota_{t+1}$	-2.24	3.78	0.32	-9.22	-0.21	0.72	-0.20	2.43	-0.20	0.09	-0.01	0.35
	(3.15)	(6.64)	(2.80)	(0.93)	(0.71)	(1.22)	(0.49)	(2.89)	(2.67)	(3.05)	(1.27)	
h_{t+1}	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
	(0.52)	(4.18)	(2.33)	(446.0)	(0.65)	(2.05)	(0.32)	(2.76)	(0.87)	(3.94)	(0.50)	
Δwp_{t+1}	0.13	0.05	-0.02	-6.90	-0.17	-0.09	-0.06	-0.12	0.03	0.01	0.00	0.13
	(0.82)	(0.42)	(0.70)	(3.13)	(2.64)	(0.69)	(0.62)	(0.66)	(2.01)	(2.08)	(0.78)	
π_{t+1}	0.06	0.00	-0.01	-0.20	0.02	0.78	0.06	-0.10	-0.01	0.00	0.00	0.77
	(0.95)	(0.11)	(1.27)	(0.24)	(0.62)	(16.0)	(1.69)	(1.37)	(0.88)	(0.92)	(0.25)	
$r_{1,t+1}$	0.05	0.06	0.00	0.35	0.00	0.12	0.92	0.03	-0.02	0.00	0.00	0.95
	(1.13)	(1.78)	(0.50)	(0.60)	(0.06)	(3.53)	(38.4)	(0.64)	(3.84)	(1.32)	(0.33)	
$sp_{b,t+1}$	-0.01	-0.05	0.00	-0.34	0.00	-0.06	0.03	0.90	0.01	0.00	0.00	0.81
	(0.39)	(1.74)	(0.37)	(0.68)	(0.17)	(1.89)	(1.27)	(20.9)	(2.65)	(0.04)	(0.75)	
$x_{b,t+1}$	-0.92	-0.36	0.16	0.42	0.03	-1.80	1.31	2.74	0.20	-0.07	0.00	0.23
	(1.50)	(0.72)	(1.60)	(0.05)	(0.12)	(3.52)	(3.71)	(3.77)	(3.12)	(2.72)	(0.39)	
$\rho_{s,t+1}$	0.74	-0.77	-0.14	16.45	0.22	-0.91	-0.02	0.93	0.32	0.04	0.07	0.10
	(0.49)	(0.64)	(0.59)	(0.78)	(0.35)	(0.72)	(0.02)	(0.52)	(2.06)	(0.55)	(3.50)	
dpr_{t+1}	-0.24	0.41	0.06	-70.06	-0.96	3.72	-0.14	8.16	-0.45	0.07	0.88	0.90
	(0.12)	(0.26)	(0.18)	(2.52)	(1.15)	(2.25)	(0.12)	(3.46)	(2.18)	(0.78)	(33.1)	
Panel B: Correlation matrix of residuals												
Δy_t	1	0.63	0.75	0.52	-0.01	-0.16	0.21	-0.09	-0.25	0.10	0.25	
Δc_t	–	1	0.17	0.33	0.15	-0.18	0.16	-0.09	-0.16	0.14	0.04	
$\Delta \iota_t$	–	–	1	0.53	-0.08	0.04	0.19	-0.07	-0.23	0.00	0.31	
Δh_t	–	–	–	1	-0.06	0.11	0.27	-0.12	-0.31	0.04	0.15	
Δwp_t	–	–	–	–	1	-0.14	-0.06	0.04	0.03	0.04	-0.11	
π_t	–	–	–	–	–	1	0.13	-0.09	-0.11	-0.14	0.09	
$r_{1,t}$	–	–	–	–	–	–	1	-0.84	-0.51	-0.11	0.05	
$sp_{b,t}$	–	–	–	–	–	–	–	1	-0.02	0.12	0.00	
$x_{b,t}$	–	–	–	–	–	–	–	–	1	-0.01	-0.06	
$\rho_{s,t}$	–	–	–	–	–	–	–	–	–	1	-0.65	
dpr_t	–	–	–	–	–	–	–	–	–	–	1	

Note: The table reports the parameter estimates (Panel A) and the correlation matrix of residuals (Panel B) for the unrestricted VAR(1) model. Numbers in parentheses represent the t-stat of the parameter estimates. We use the notations $sp_{b,t} = y_{b,t} - r_{1,t}$, $x_{b,t} = r_{b,t} - r_{1,t}$, $x_{s,t} = r_{s,t} - r_{1,t}$, and $dpr_t = dp_t - vp_t$.

Table A5: Parameter estimates of the VAR model with financial variables only

	$r_{1,t}$	$x_{s,t}$	$x_{b,t}$	$r_{1,t}^{(n)}$	dpr_t	$sp_{b,t}$	R^2
Panel A: Parameter estimates							
$r_{1,t+1}$	0.69	0.00	-0.02	0.19	0.00	0.18	0.75
	(13.68)	(0.01)	(2.22)	(4.39)	(2.41)	(2.45)	
$x_{s,t+1}$	-0.81	0.04	0.35	0.29	0.00	2.05	0.04
	(0.69)	(0.61)	(2.22)	(0.29)	(0.66)	(1.22)	
$x_{b,t+1}$	1.25	-0.09	0.22	-0.26	0.00	2.46	0.19
	(2.71)	(3.33)	(3.62)	(0.66)	(1.04)	(3.72)	
$r_{1,t+1}^{(n)}$	-0.07	0.00	-0.02	1.03	0.00	0.09	0.95
	(2.32)	(1.85)	(4.88)	(37.63)	(0.70)	(1.90)	
dpr_{t+1}	-2.47	0.08	-0.46	1.62	1.01	5.18	0.89
	(1.58)	(0.88)	(2.21)	(1.22)	(235.70)	(2.30)	
$sp_{b,t+1}$	0.03	0.00	0.01	-0.02	0.00	0.85	0.80
	(1.03)	(0.38)	(3.66)	(0.88)	(1.62)	(21.5)	
Panel B: Correlation matrix of residuals							
$r_{1,t+1}$	1	-0.01	-0.22	0.50	-0.04	-0.45	
$x_{s,t+1}$	–	1	-0.01	-0.12	-0.67	0.13	
$x_{b,t+1}$	–	–	1	-0.53	-0.05	0.02	
$r_{1,t+1}^{(n)}$	–	–	–	1	0.04	-0.85	
dpr_{t+1}	–	–	–	–	1	0.01	
$sp_{b,t+1}$	–	–	–	–	–	1	

Note: The table reports the parameter estimates (Panel A) and the correlation matrix of residuals (Panel B) for the unrestricted VAR(1) model. Numbers in parentheses represent the t-stat of the parameter estimates. We use the notations $sp_{b,t} = y_{b,t} - r_{1,t}$, $x_{b,t} = r_{b,t} - r_{1,t}$, $x_{s,t} = r_{s,t} - r_{1,t}$, and $dpr_t = dp_t - vp_t$.

C Dynamics of Parameter Estimates

Figure A1: Parameter dynamics: Structural parameters

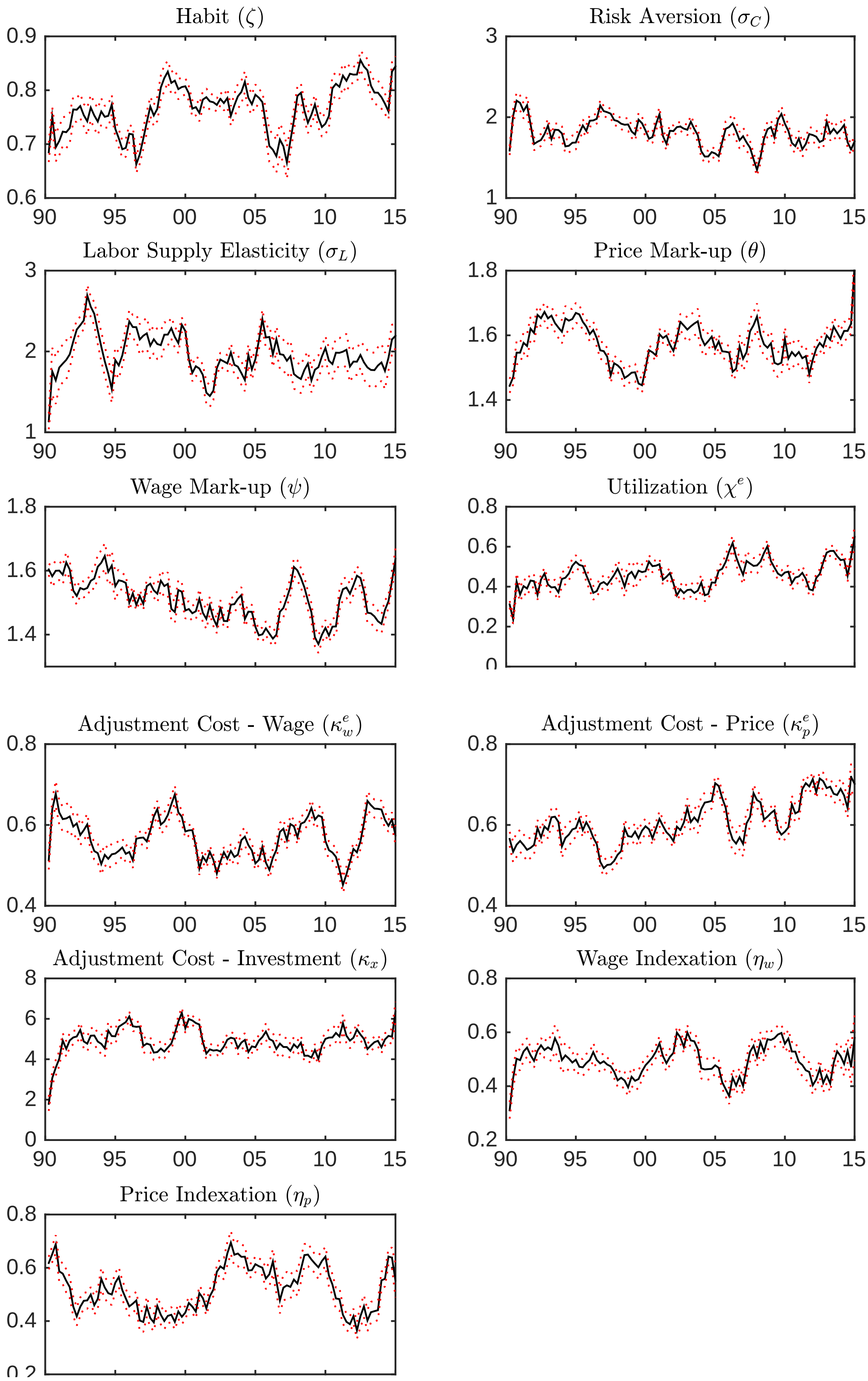


Figure A2: Parameter dynamics: Monetary policy

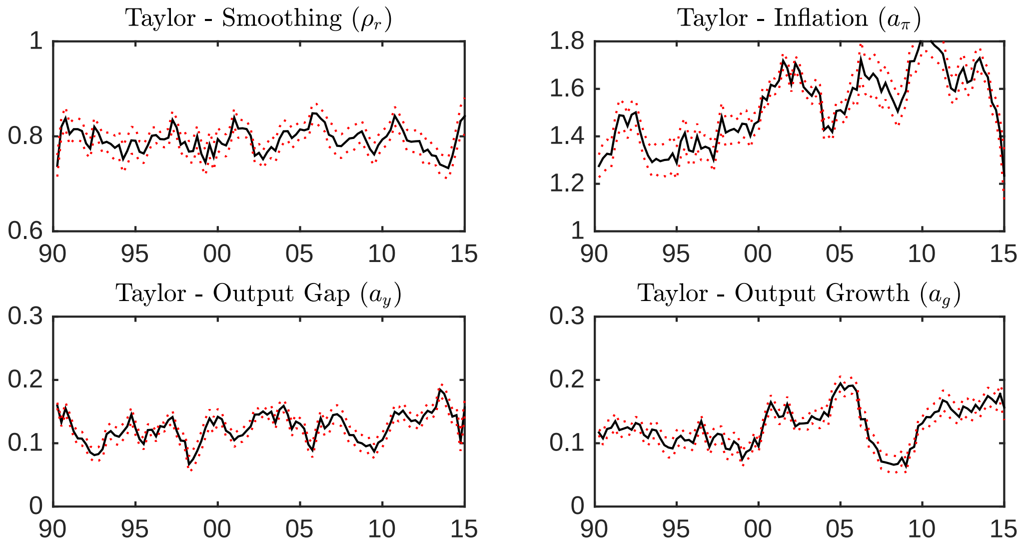


Figure A3: Parameter dynamics: Autoregressive terms

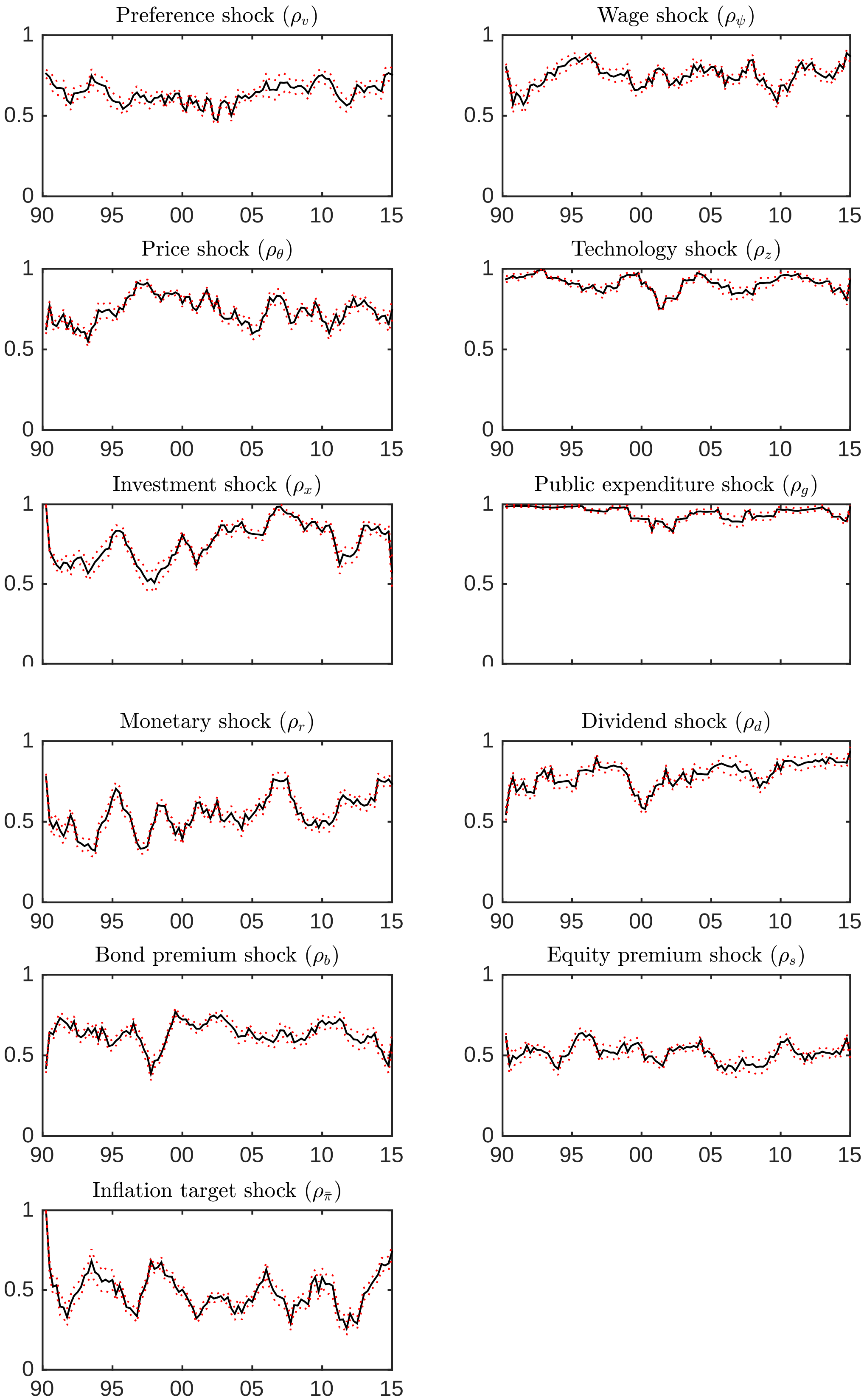


Figure A4: Parameter dynamics: Moving average terms

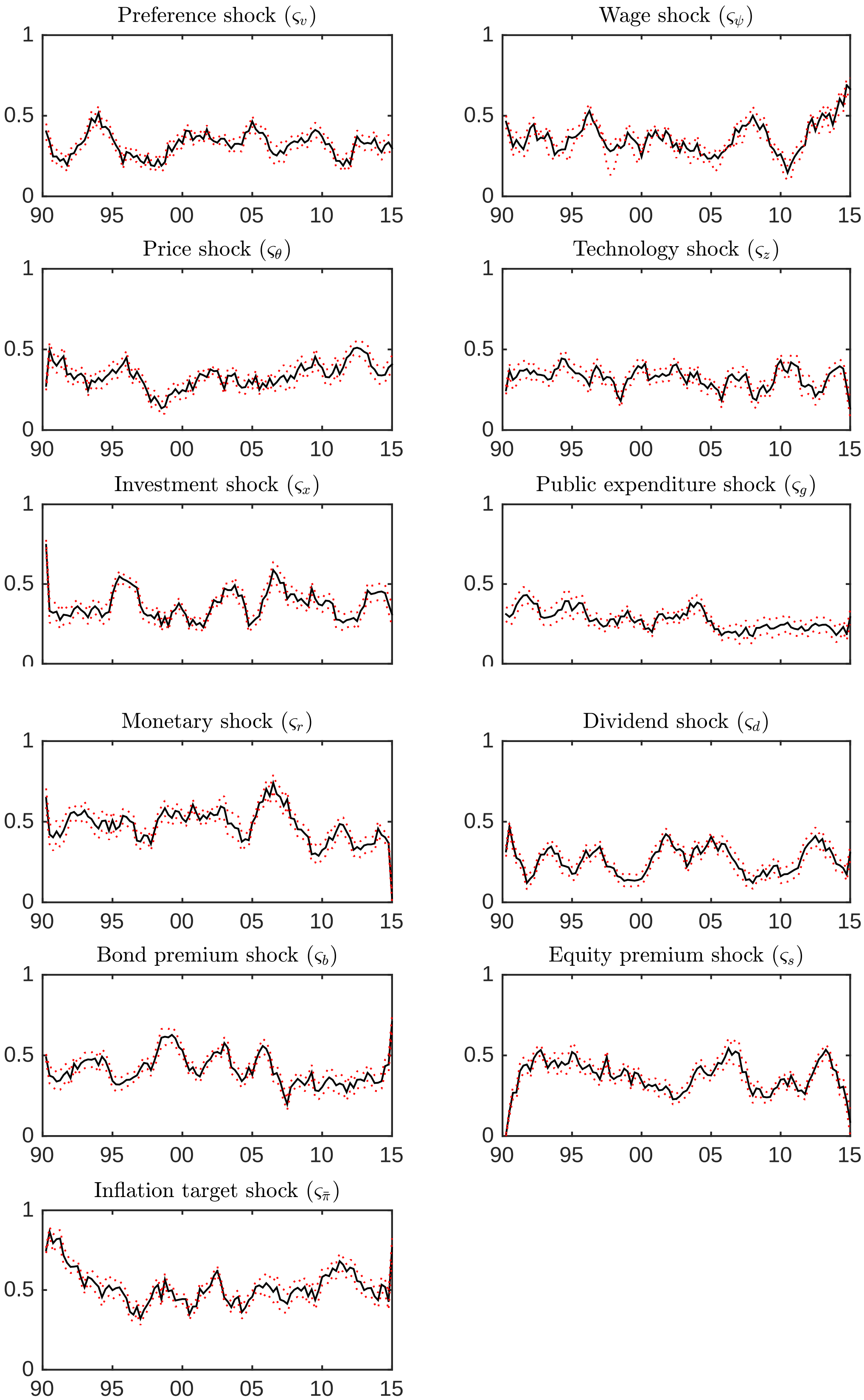


Figure A5: Parameter dynamics: Spillover shock to bond premium

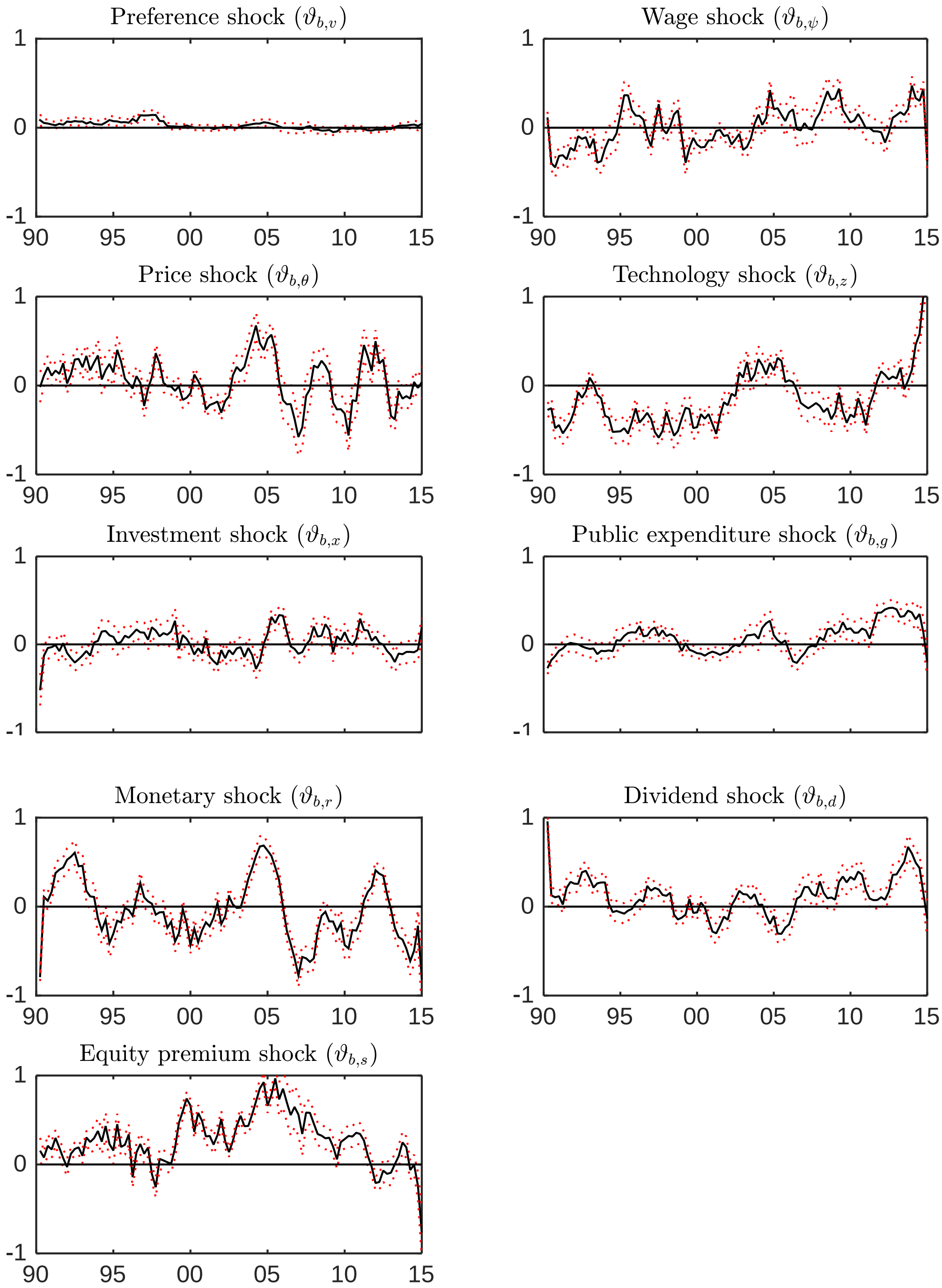


Figure A6: Parameter dynamics: Spillover shock to stock premium

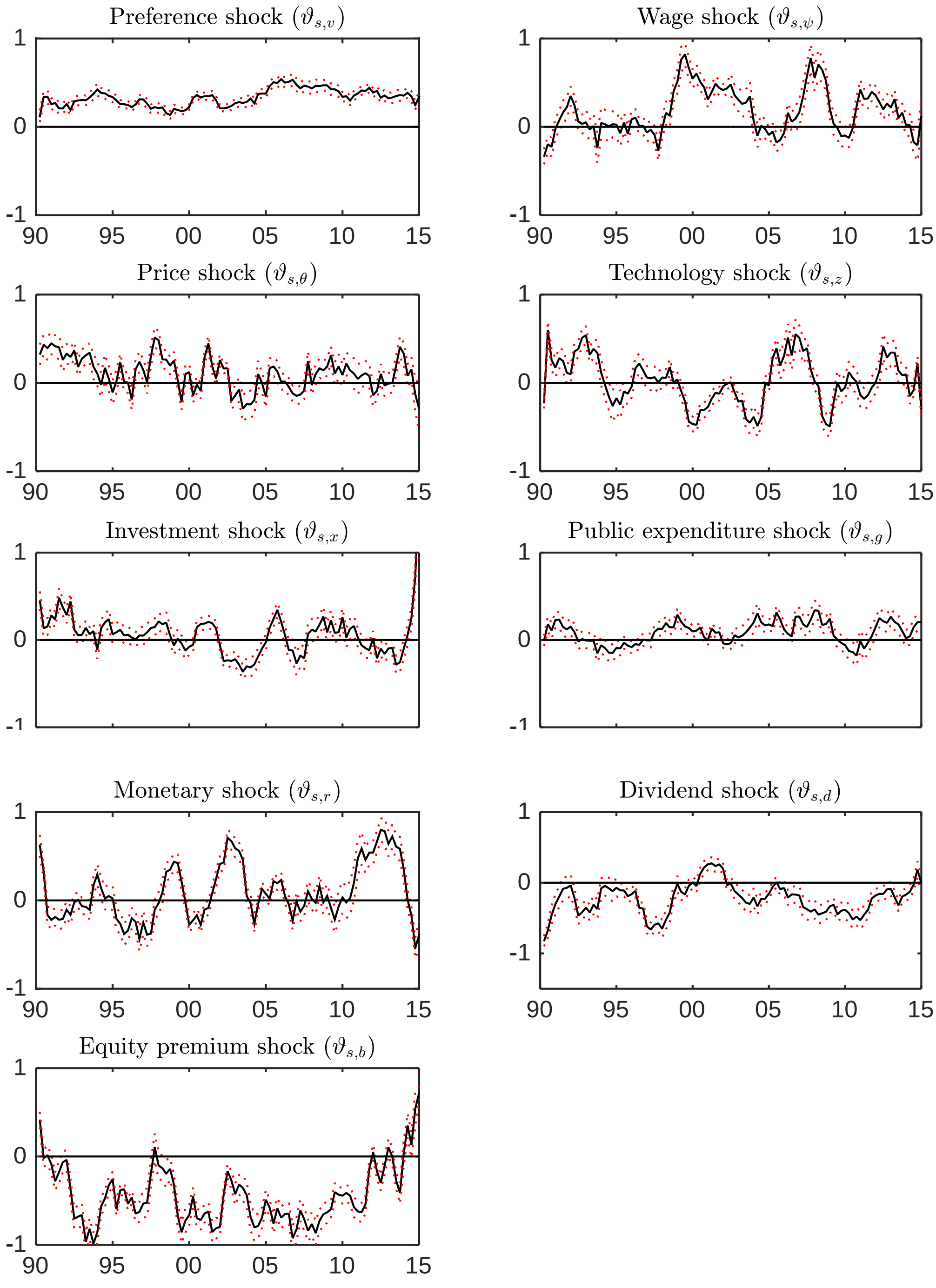


Figure A7: Parameter dynamics: Std dev. of shocks